



**ABCP 2026 Annual Conference: *Artificial Intelligence and Cyber Security Track***

# **GNSS Interference Detection and Mitigation Technologies**

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Cavendish Laboratory, University of Cambridge, UK**

# Outline

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- **Satellite Navigation Countermeasures**
- **GNSS Interference Detection based on TF Analyses**
- **GNSS Interference Detection based on Hough Transform**
- **GNSS Anti-Interference based on FrFT**
- **GNSS Anti-Interference based on FrFT & Notch Filtering**



## Industry empowerment

# Outline

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- GNSS Interference Detection based on Hough Transform
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- GNSS Anti-Interference based on FrFT & Notch Filtering



# GNSS Interference Detection using Choi-Williams Transform

## Choi-Williams Transform (CWT)

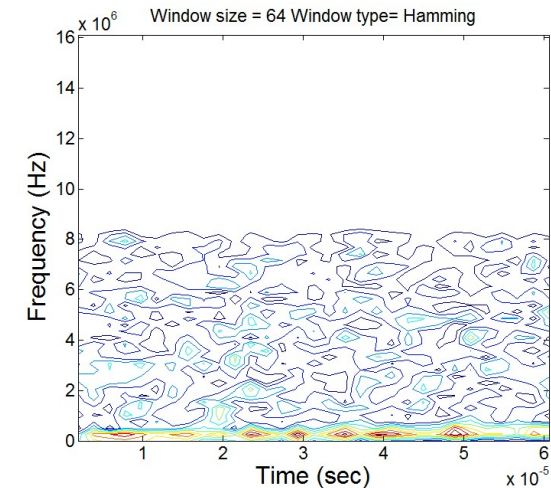
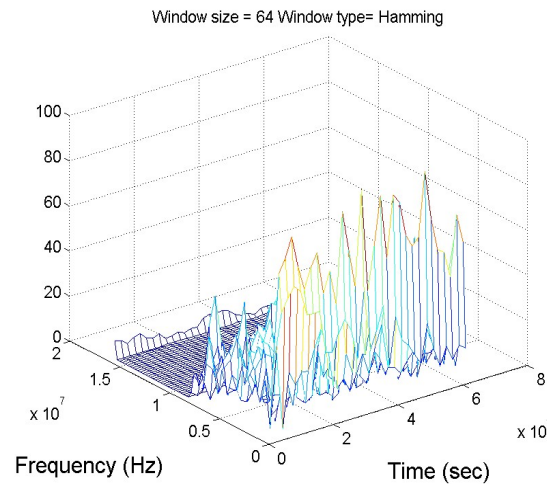
### ■ Spectrogram

- The spectrogram is a visual representation of the spectrum of frequencies of a signal as it varies with time, which is computed using the Short-Time Fourier Transform (STFT).

$$\text{Spectrogram}(t, \omega) = \left| \int_{-\infty}^{\infty} y(\tau) \gamma(\tau - t) e^{-j\omega\tau} d\tau \right|^2$$

- For a signal  $y(t)$ , define its temporal standard deviation as  $\Delta t$  and its spectral standard deviation as  $\Delta f$ . The uncertainty principle can be expressed as the uncertainty principle states:  $\Delta t \cdot \Delta f \geq \frac{1}{4\pi}$
- The spectrogram presents the time-frequency (TF) resolution trade off problem, providing poor localization properties in the TF plane.

*The spectrogram of Galileo E1 OS signal with sweep interference by using a Hamming window of length  $L = 64$  samples.  $C/N_0 = 38 \text{ dB-Hz}$ ,  $JNR = -10 \text{ dB}$ .*

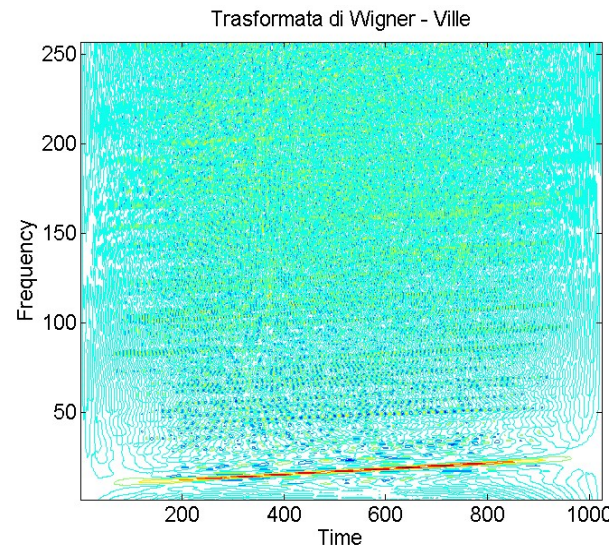
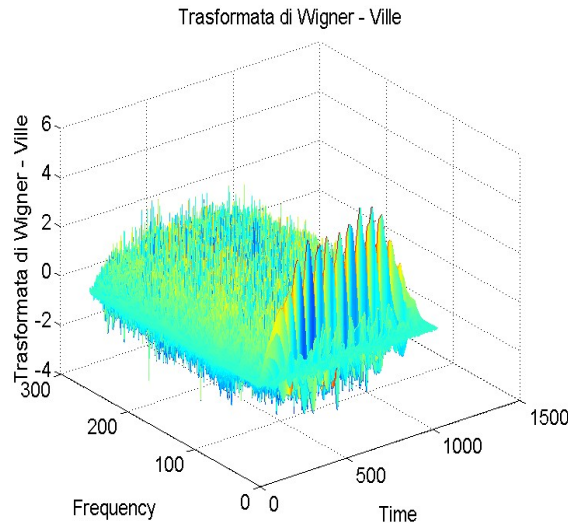


# GNSS Interference Detection using Choi-Williams Transform

## ■ Wigner-Ville Distribution (WVD)

The **Wigner-Ville Distribution (WVD)** is a powerful tool in time-frequency (TF) analysis that provides a joint representation of a signal in both time and frequency domains, but it can introduce **cross-term interference** due to its quadratic nature.

$$WVD_y(t, f) = \int_{-\infty}^{\infty} y\left(t + \frac{\tau}{2}\right) y^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau$$



The WVD shows improved TF localization properties but **presents very severe cross interfering terms problems** due to interaction of different frequency components.

*The Wigner-Ville distribution of Galileo E1 OS signal with sweep interference.  $C/N_0 = 38 \text{ dB-Hz}$ ,  $JNR = -10 \text{ dB-Hz}$ .*

# GNSS Interference Detection using Choi-Williams Transform

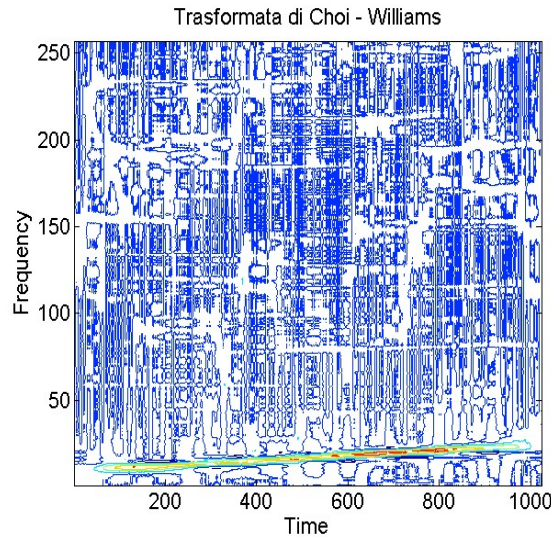
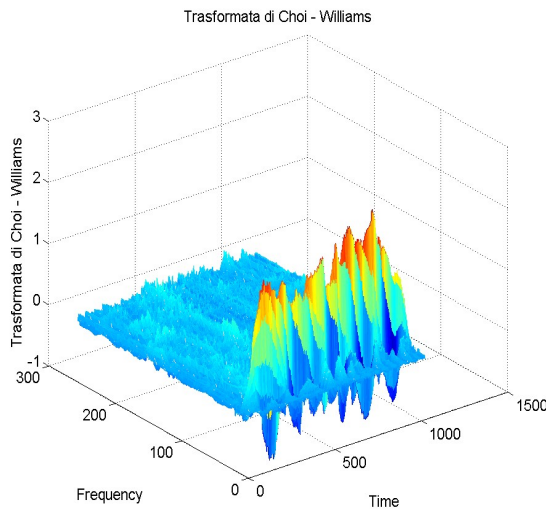
## ■ Choi-Williams Transform (CWT)

- To mitigate cross-term interference in the Wigner-Ville Distribution (WVD), **the Choi-Williams kernel function** can be incorporated into Cohen's class of time-frequency distributions, yielding the Choi-Williams Transform (CWT).

$$CWT_y(t, f) = \iint_{-\infty}^{\infty} \varphi(\theta, \tau) AF_y(\theta, \tau) e^{-j2\pi(f\tau - \theta t)} d\theta d\tau$$

where:  $\varphi(\theta, \tau)$  is the Choi-Williams kernel:  $\varphi(\theta, \tau) = e^{-\frac{(\theta\tau)^2}{\sigma}}$

$AF_y(\theta, \tau)$  is the ambiguity function of  $y(t)$ :  $AF_y(\theta, \tau) = \int_{-\infty}^{\infty} y\left(t + \frac{\tau}{2}\right) y^*\left(t - \frac{\tau}{2}\right) e^{j2\pi\theta t} dt$



The CWT partially mitigates the cross terms in the TF plane by using an effective attenuating Choi-Williams kernel function.

- Kewen Sun\*, Wei Liu, Huamin Xu, Yang Dongkai, Interference Detection based on Time-Frequency Analysis for GNSS Receivers, *ION GNSS+ 2013 Conference*, Nashville, Tennessee, USA, September 16-20, 2013

*The CWT of Galileo E1 OS signal with sweep interference.  $C/N_0 = 38 \text{ dB-Hz}$ ,  $JNR = -10 \text{ dB-Hz}$ .*

# GNSS Interference Detection using Joint Hybrid TF Analysis

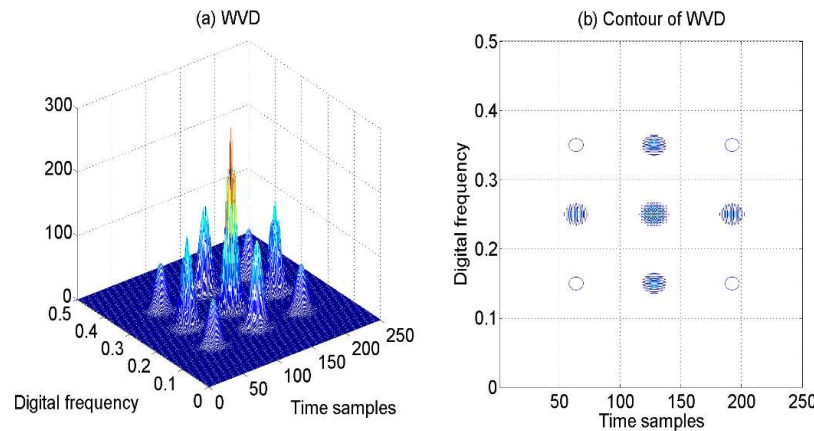
## Joint Hybrid TF Analysis based on SPWVD and CWT

### Pseudo Wigner-Ville Distribution (PWVD)

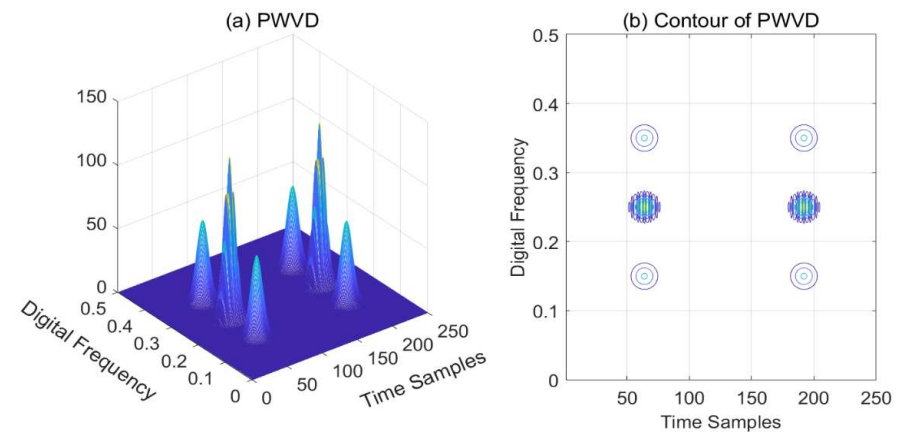
- The Pseudo Wigner-Ville Distribution (PWVD) is a modified version of the standard Wigner-Ville Distribution (WVD) that applies a **smoothing window in the frequency domain** to reduce interference terms while maintaining reasonable time-frequency resolution.

$$PW_y(t, f) = \int_{-\infty}^{\infty} h(\tau) y\left(t + \frac{\tau}{2}\right) y^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau$$

- The PWVD introduces a symmetric window  $h(\tau)$  to limit the integration range, reducing cross-terms, but may still exhibit some residual interference.



The WVD of four Gaussian components' signal.



The PWVD of four Gaussian components' signal.

# GNSS Interference Detection using Joint Hybrid TF Analysis

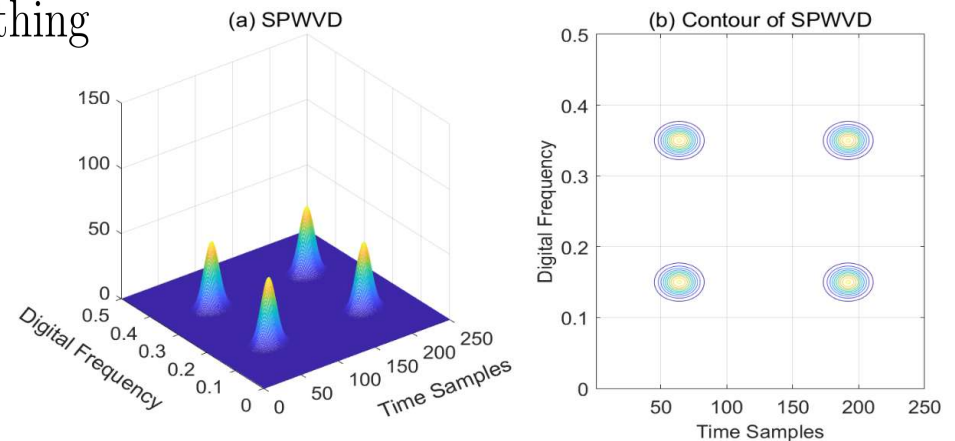
## ■ Smoothed Pseudo Wigner-Ville Distribution (SPWVD)

- Since the cross-terms with the WVD are strongly oscillating, the most effective way of removing such cross-term interference is to apply **two-dimensional low-pass filtering** in the ambiguity function domain, developing the smoothed version of pseudo WVD, that is Smoothed Pseudo Wigner-Ville Distribution (SPWVD).
- The SPWVD is a time-frequency analysis tool designed to reduce the interference terms present in the standard WVD while maintaining good resolution. **It applies smoothing windows separately to the time and frequency domains.**

$$\text{SPWVD}_y(t, f) = \int_{-\infty}^{\infty} h(\tau) \left[ \int_{-\infty}^{\infty} g(u - t) y(u + \tau/2) y^*(u - \tau/2) du \right] e^{-j2\pi f\tau} d\tau$$

SPWVD = Time Windowing (PWVD) + Frequency Smoothing

- Although the adoption of the smoothing window in SPWVD is beneficial to suppress the cross-terms in the TF plane, **it smears localized components, leading to less accurate TF localization of the auto-terms of the signal, as compared to the WVD approach.**



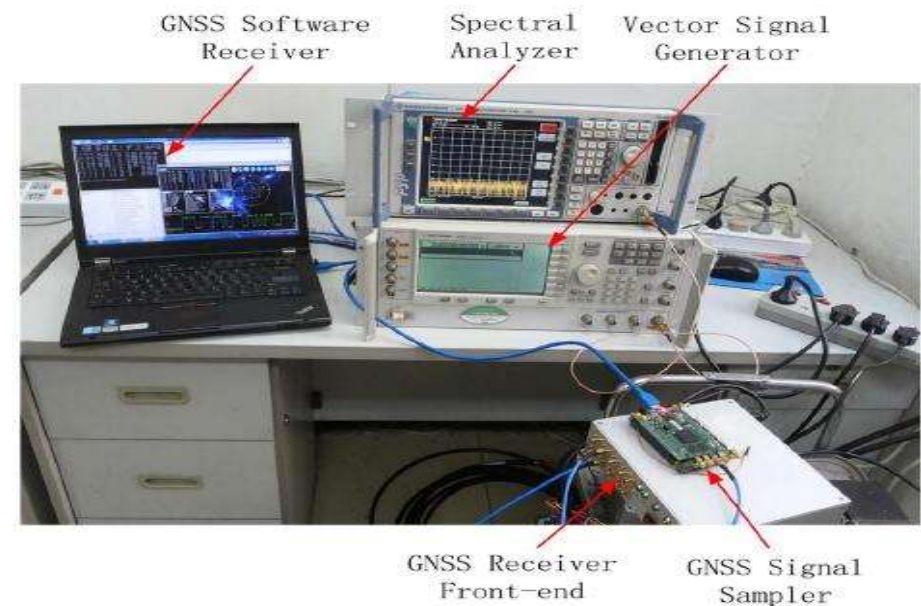
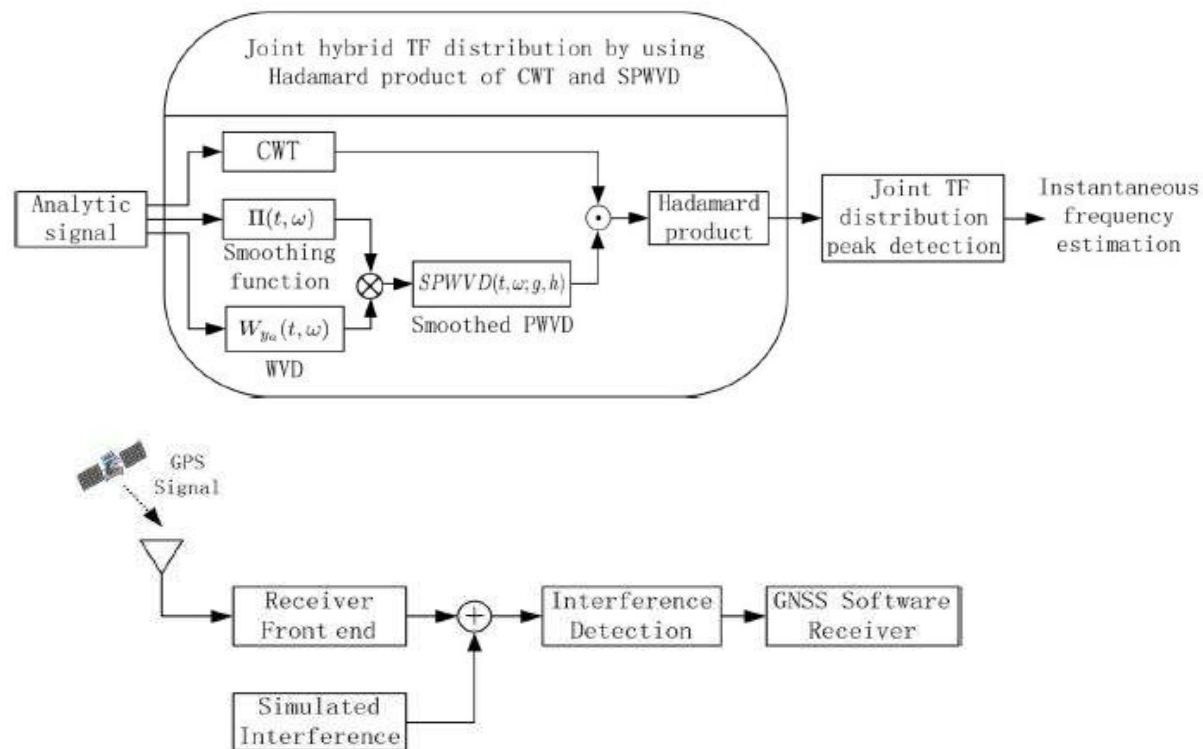
*The SPWVD of four Gaussian components' signal.*

# GNSS Interference Detection using Joint Hybrid TF Analysis

## ■ Hadamard Product of CWT and SPWVD

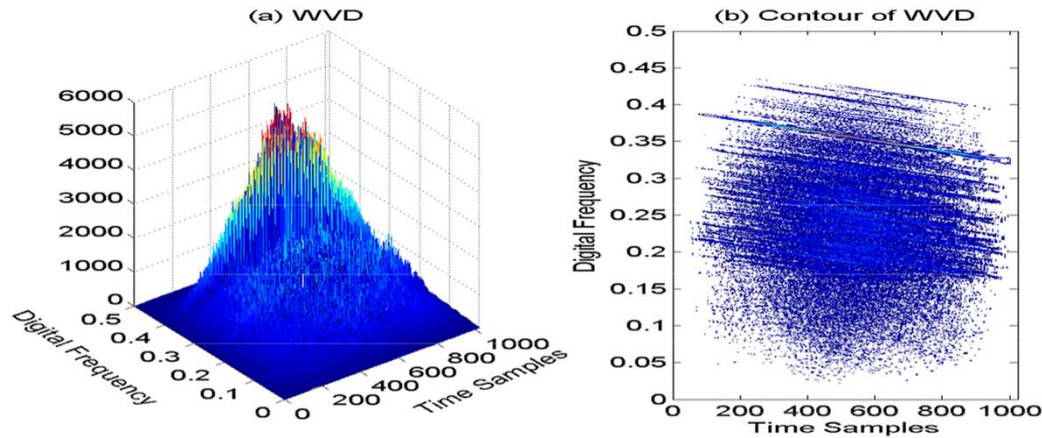
- To eliminate the cross-terms present in the quadratic TF distribution and to preserve good time and frequency resolutions at the same time, **a novel joint hybrid TF distribution based on the Hadamard product of CWT and SPWVD** has been proposed in interference detection for GNSS receivers.

$$M(t, \omega) = CWT(t, \omega) \odot SPWVD(t, \omega)$$

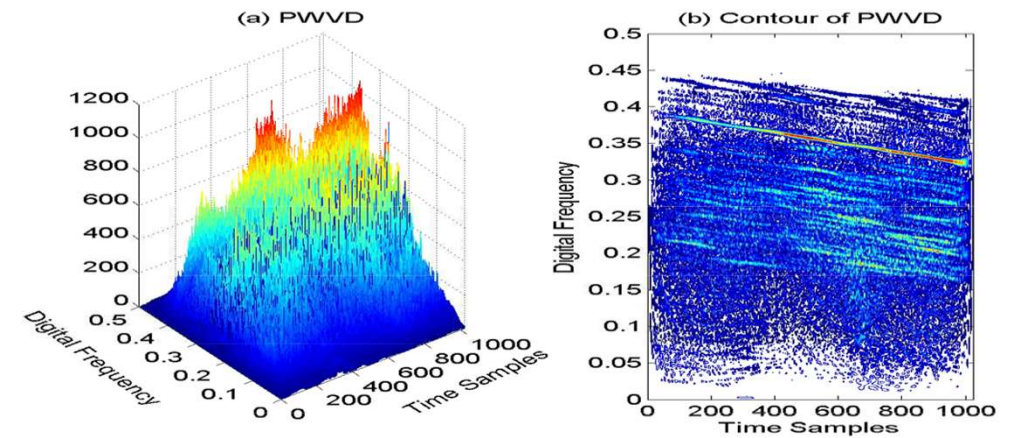


# GNSS Interference Detection using Joint Hybrid TF Analysis

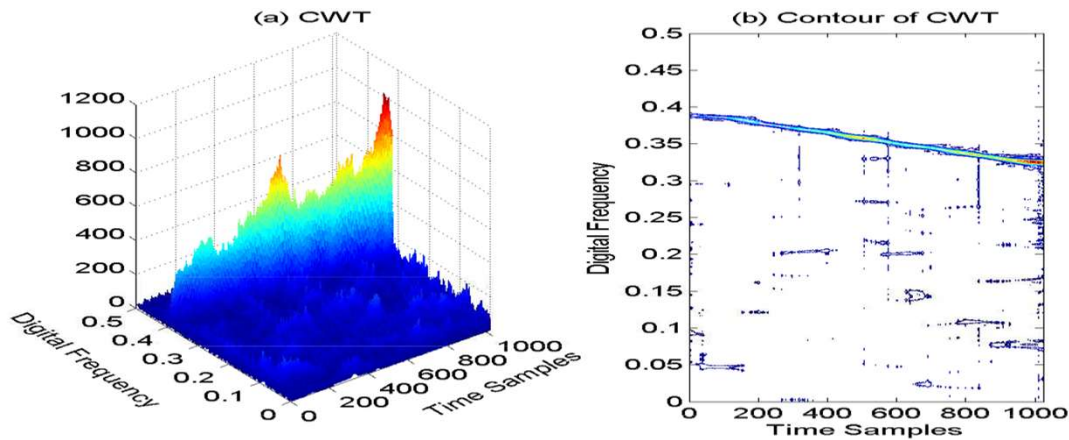
## ■ Sweep Interference Detection Results



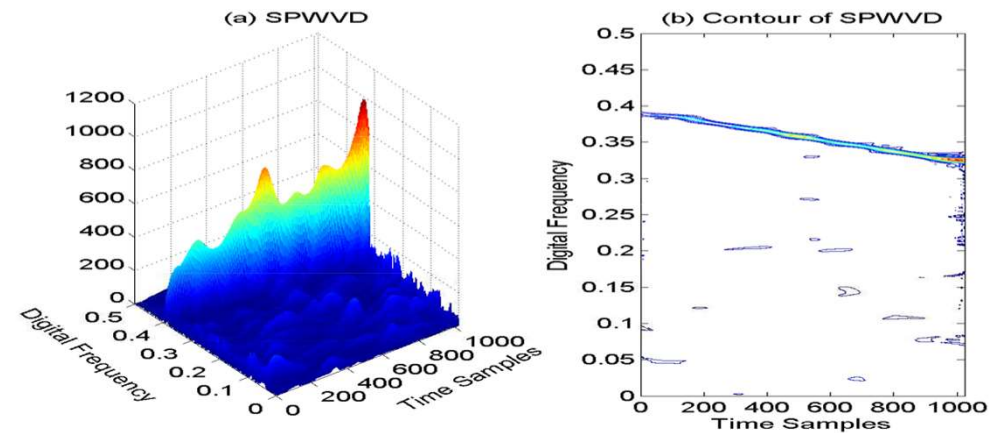
WVD of GPS L1-C/A signal with sweep interference.  $C/N_0 = 46 \text{ dB-Hz}$ ,  $JNR = -8 \text{ dB}$ .



PWVD of GPS L1-C/A signal with sweep interference.



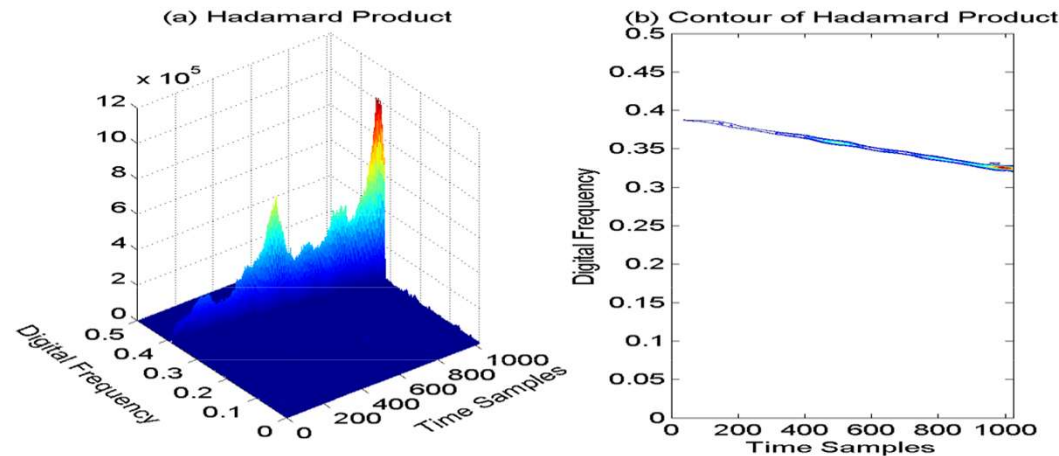
CWT of GPS L1-C/A signal with sweep interference.



SPWVD of GPS L1-C/A signal with sweep interference.

# GNSS Interference Detection using Joint Hybrid TF Analysis

- The cross-terms are partially mitigated by the Choi-Williams kernel function, and the peaks of the CWT are clearly observed denoting the GNSS interference contribution.
- The adoption of the smoothing window in SPWVD effectively suppress the cross-terms in the TF plane, but it smears out localized components, leading to less accurate TF localization of the auto-terms of the signal compared to the original WVD.
- **The peaks of the joint hybrid TF distribution emerge as a line representing the contribution of the sweep interference while its values remain zero in the other area of the TF plane.**

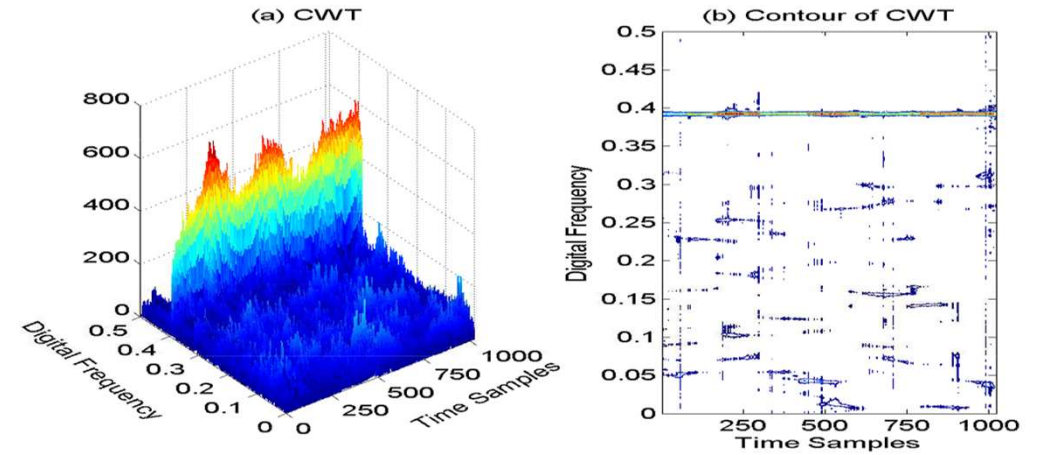
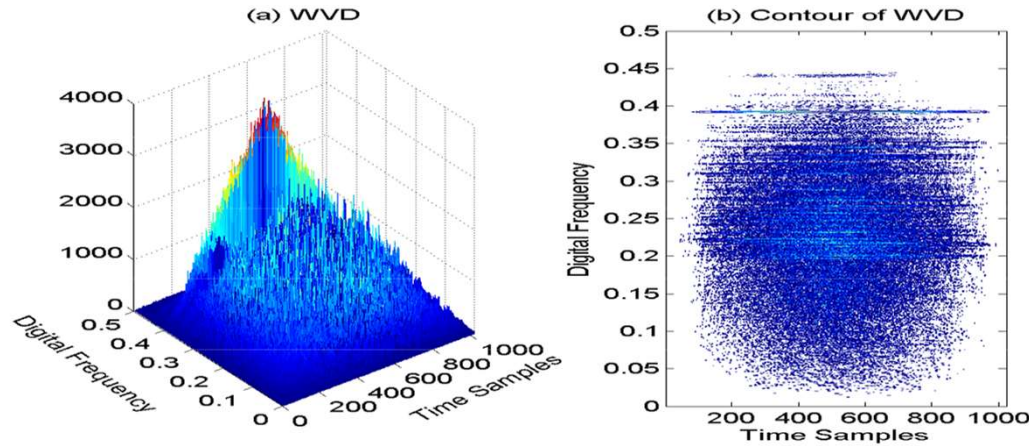


*Hadamard Product of CWT and SPWVD for GPS L1-C/A signal with sweep interference.  $C/N_0 = 46 \text{ dB-Hz}$ ,  $JNR = -8 \text{ dB}$ .*

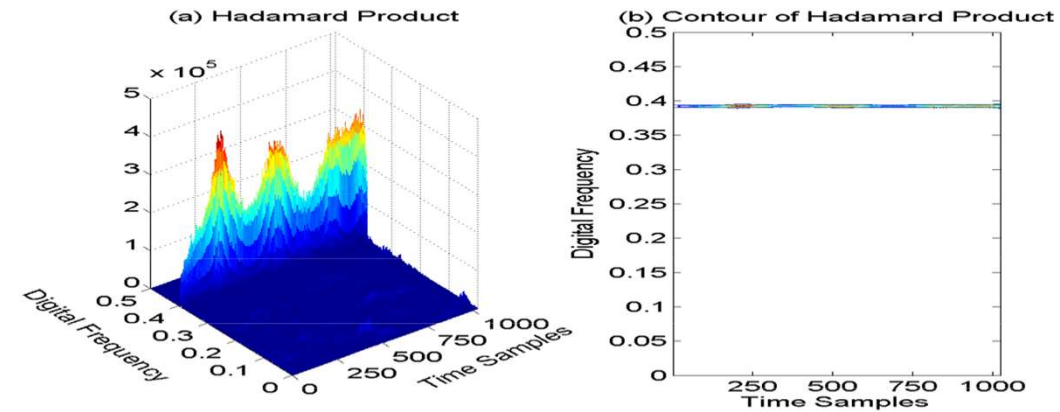
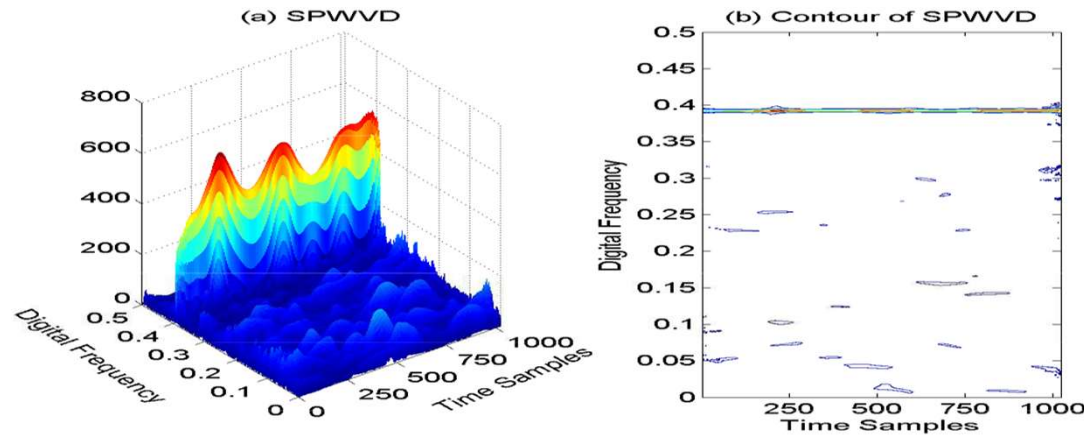
- Kewen Sun\*, Min Zhang, Dongkai Yang, A New Interference Detection Method based on Hybrid Time-Frequency Distribution for GNSS Receivers, *IEEE Transactions on Vehicular Technology*, 2016, 65(11): 9057-9071

# GNSS Interference Detection using Joint Hybrid TF Analysis

## Continuous Wave Interference Detection Results



WVD of GPS L1-C/A signal in the presence of the CWI.  $C/N_0 = 46 \text{ dB-Hz}$ ,  $JNR = -8 \text{ dB}$ . CWT of GPS L1-C/A signal in the presence of the CWI.  $C/N_0 = 46 \text{ dB-Hz}$ ,  $JNR = -8 \text{ dB}$ .



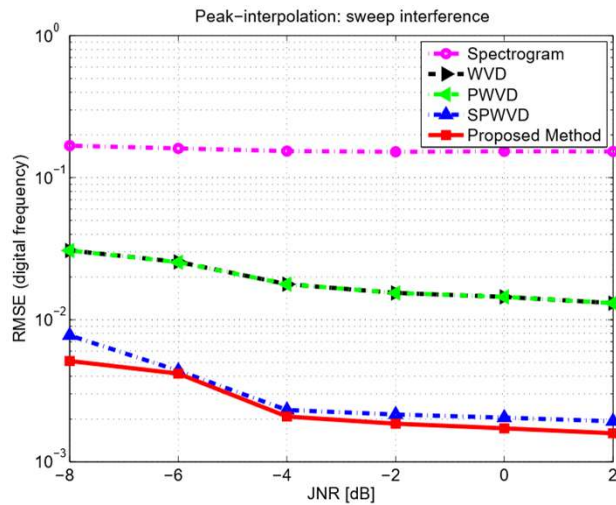
SPWVD of GPS L1-C/A signal in the presence of the CWI.  $C/N_0 = 46 \text{ dB-Hz}$ ,  $JNR = -8 \text{ dB}$ .

Hadamard Product of CWT and SPWVD for GPS L1-C/A signal with the CWI.  $C/N_0 = 46 \text{ dB-Hz}$ ,  $JNR = -8 \text{ dB}$ .

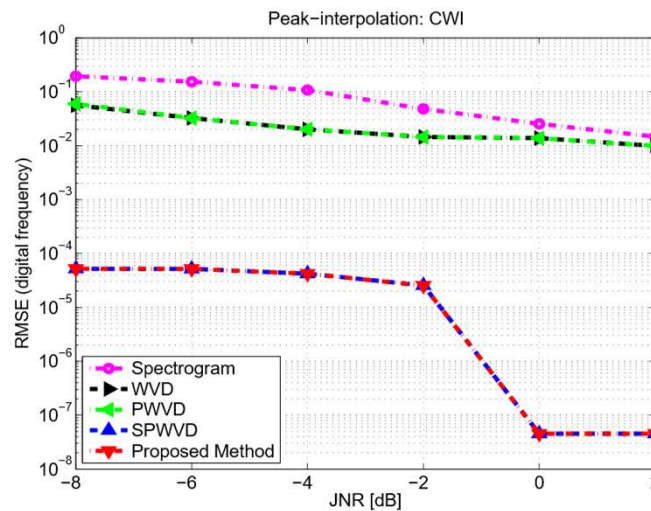
# GNSS Interference Detection using Joint Hybrid TF Analysis

- The cross-interfering terms are effectively suppressed by using the SPWVD, but deteriorated TF localization properties are obtained.
- The proposed joint hybrid TF analysis method completely eliminates the cross-terms present in the TF distributions, presenting good readability in the TF plane and preserving satisfactory TF resolutions.**

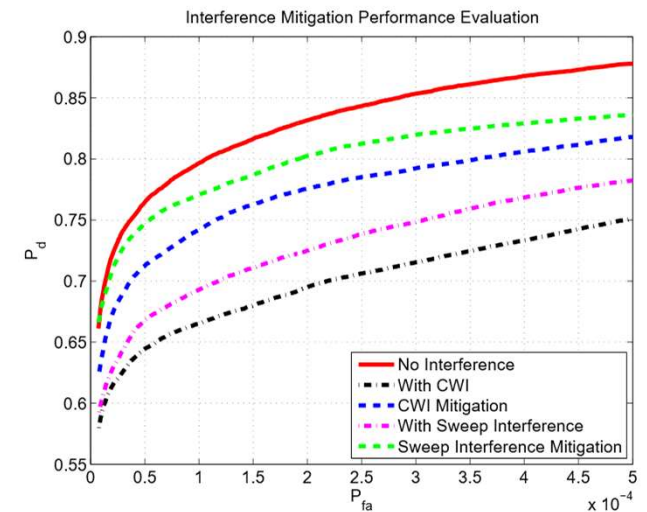
## ■ Statistical Performance Analysis



RMSE of the frequency estimation for the **sweep interference** present in the received GPS L1-C/A signal.



RMSE of the frequency estimation for the **CWI** present in the received GPS L1-C/A signal.



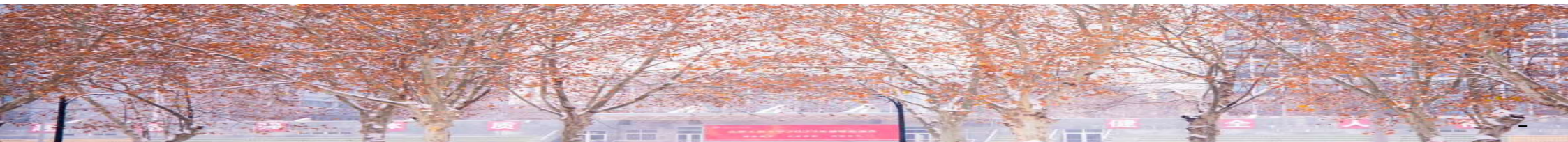
ROC curves in different signal scenarios. The coherent integration time is 2 ms,  $C/N_0 = 37$  dB-Hz,  $JNR = 15$  dB.

- GNSS interference estimation precision is nearly improved by 3 orders of magnitude; interference detection performance is improved by more than 15% with the proposed method.**

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- GNSS Anti-Interference based on FrFT & Notch Filtering

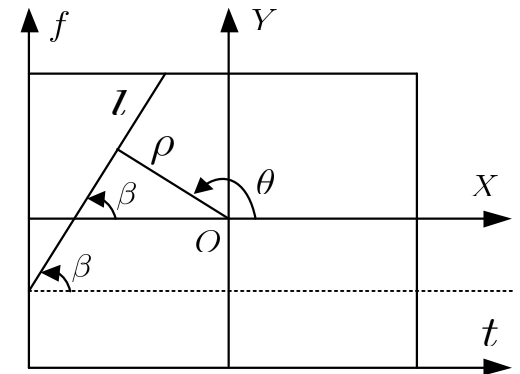


# Rearranged Wavelet–Hough Transform Method

## Wigner-Hough Transform (WHT)

- Hough transform can be used to detect the line of frequency modulation law for a given signal in the TF image space.

$$\mathcal{WHT}_y(\rho, \theta) = \iint_{-\infty}^{\infty} W_y(t, f) \delta(\rho - t \cos \theta - f \sin \theta) dt df$$



*Scheme of Hough transform*

## Hough Transform on Reassigned Scalogram

### Step 1 - Continuous Wavelet Transform (CWT)

$$W_y(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} y(t) \psi^*\left(\frac{t-b}{a}\right) dt, \quad a > 0, b \in \mathbb{R}.$$

### Step 2 - Reassignment to Time-Frequency Plane

The reassigned time  $t'(a, b)$  and frequency  $f'(a, b)$  are computed using gradients of the squared CWT modulus.

$$t'(a, b) = b + \frac{\partial \ln |W(a, b)|^2}{\partial b} \cdot \frac{a^2}{2}, \quad f'(a, b) = \frac{f_0}{a} - \frac{\partial \ln |W(a, b)|^2}{\partial (1/a)} \cdot \frac{f_0}{a^2}$$

# Rearranged Wavelet–Hough Transform Method

## Step 3 - Reassigned Wavelet Transform (RWT):

The RWT redistributes the energy of  $W(a, b)$  to the reassigned points  $(t', f')$ .

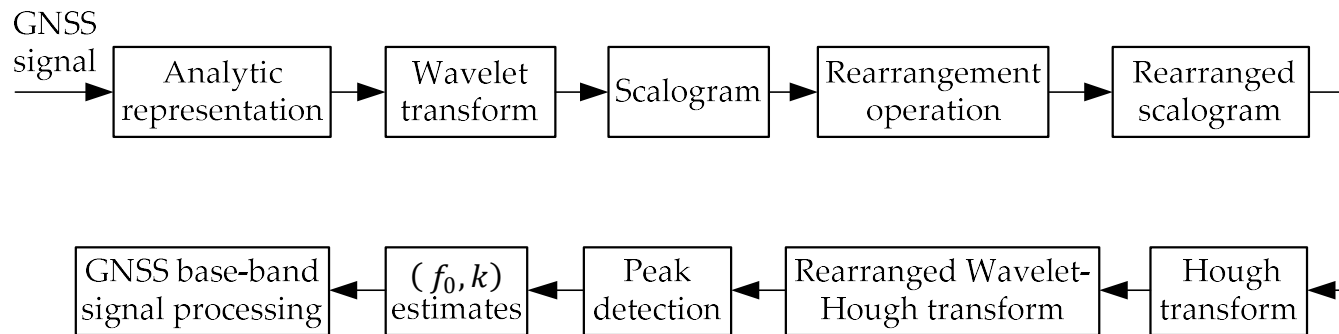
$$RWT(t, f) = \int_0^\infty \int_{-\infty}^\infty \frac{1}{a^2} |W(a, b)|^2 \cdot \delta(t - t'(a, b)) \delta(f - f'(a, b)) db da$$

## Step 4 - Hough Transform Applied to RWT

The Hough Transform accumulates energy along curves (e.g., lines) in the time-frequency plane. For detecting lines parameterized by angle (theta) and distance (rho),

$$\rho = t \cos \theta + f \sin \theta$$

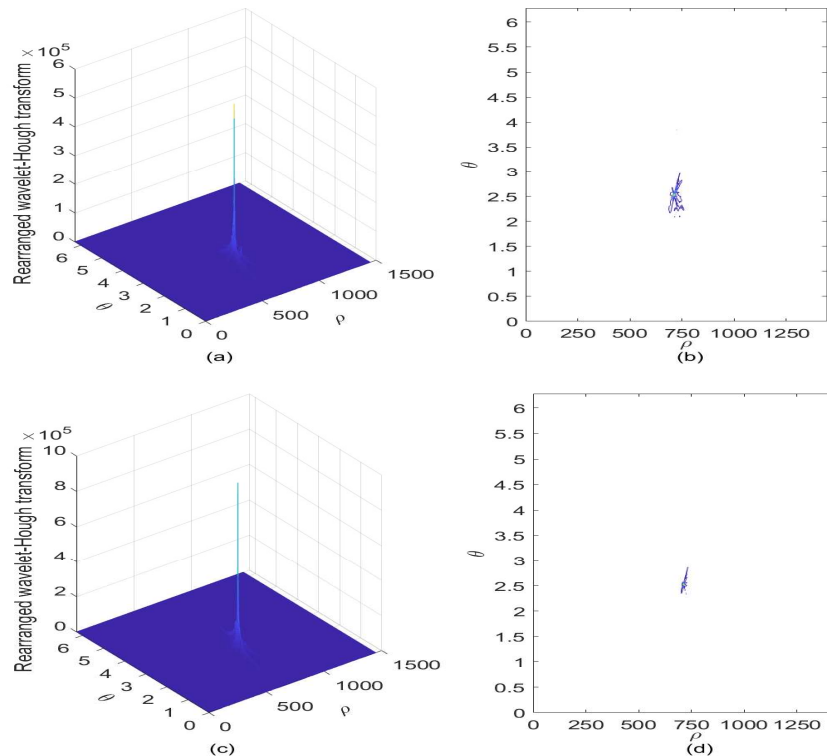
$$RWHT(\theta, \rho) = \int_{-\infty}^\infty \int_{-\infty}^\infty RWT(t, f) \cdot \delta(\rho - t \cos \theta - f \sin \theta) dt df$$



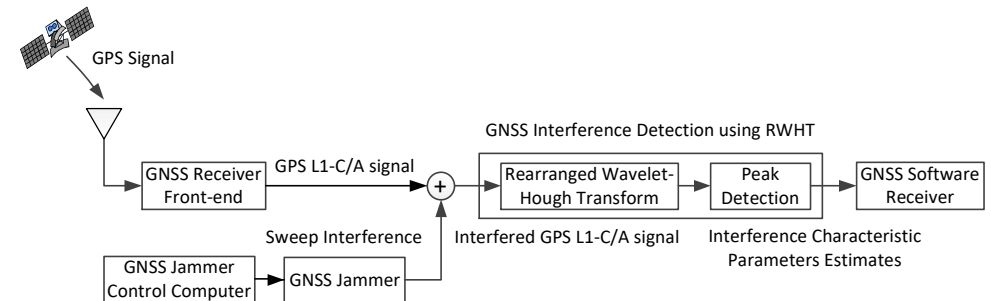
*GNSS Interference detection by using the rearranged wavelet-Hough transform for GNSS receivers.*

# Rearranged Wavelet–Hough Transform Method

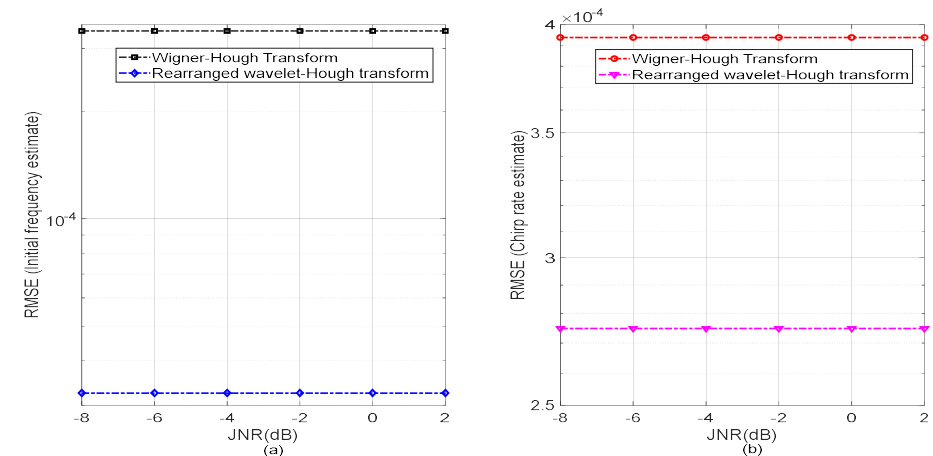
## Performance Evaluation



The rearranged Wavelet-Hough transform of the GPS L1-C/A signal in the presence of sweep interference.  $C/N_0 = 40$  dB-Hz. (a) Rearranged Wavelet-Hough transform, JNR=-8 dB; (b) Contour of rearranged Wavelet-Hough transform; (c) Rearranged Wavelet-Hough transform, JNR=-2 dB; (d) Contour of rearranged Wavelet-Hough transform.



The scheme of the GNSS interference detection test in the lab.



The RMSE of the estimated chirp rate and initial frequency of the sweep interference present in the received GPS L1-C/A signal by using Wigner-Hough transform and rearranged Wavelet-Hough transform, respectively.  $C/N_0=40$  dB-Hz. (a) RMSE of the estimated initial frequency of the sweep interference; (b) RMSE of the estimated chirp rate of the sweep interference.

- Kewen Sun\*, Tengpeng Zhang, A New GNSS Interference Detection Method Based on Rearranged Wavelet–Hough Transform, *Sensors*, 2021, 21(5):1714

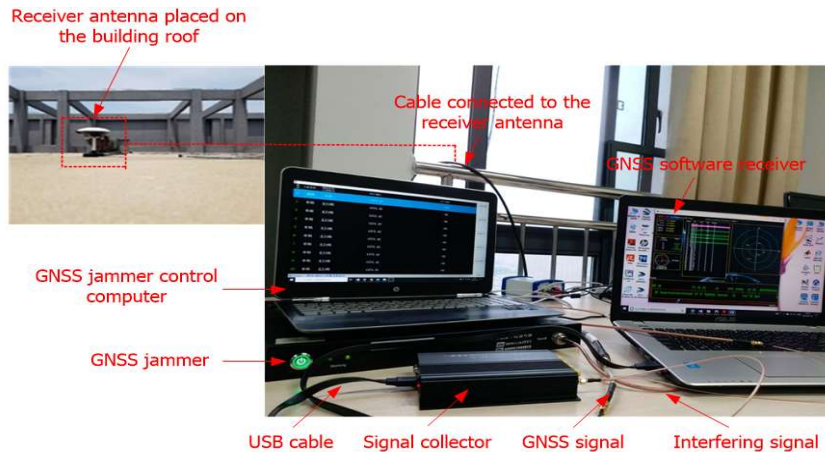
# Smoothed Pseudo WVD–Hough Transform Method

## Smoothed Pseudo Wigner-Hough Transform (SPWHT)

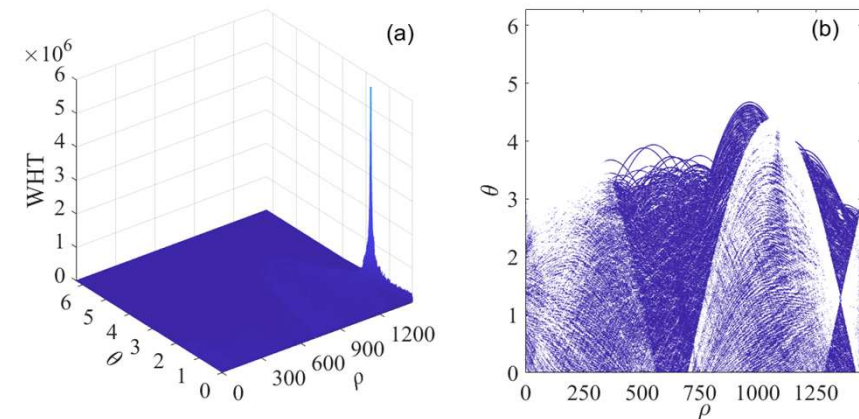
- ✓ We first define the smoothed pseudo-Wigner distribution via a 2D kernel, and then having its Hough transform.

$$W_y^\Phi(t, f) = \iint_{-\infty}^{\infty} \Phi(\tau, \nu) y\left(t + \frac{\tau}{2}\right) y^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi(f-\nu)\tau} d\tau d\nu$$
$$SPWHT_y(\rho, \theta) = \iint_{-\infty}^{\infty} W_y^\Phi(t, f) \delta(\rho - t \cos \theta - f \sin \theta) dt df$$
$$\theta \in [0, \pi), \rho \in \mathbb{R}$$

## Performance Evaluation

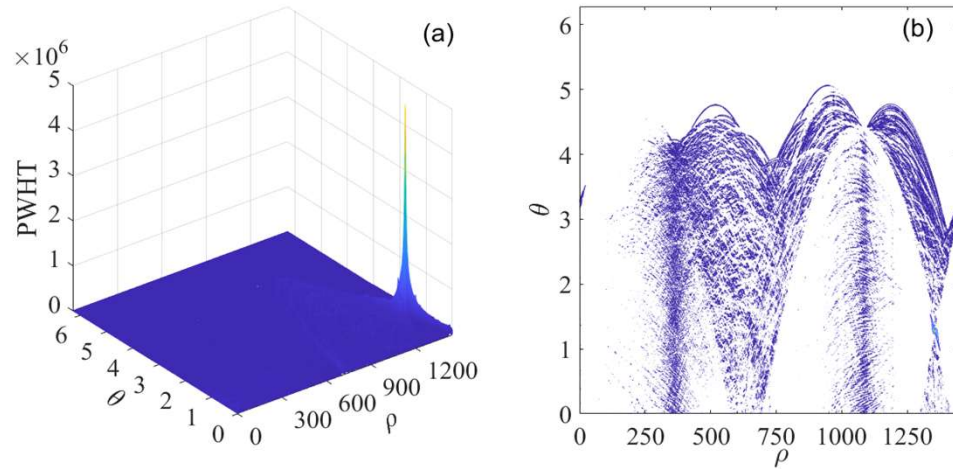


The experimental platform of the GNSS interference detection test.

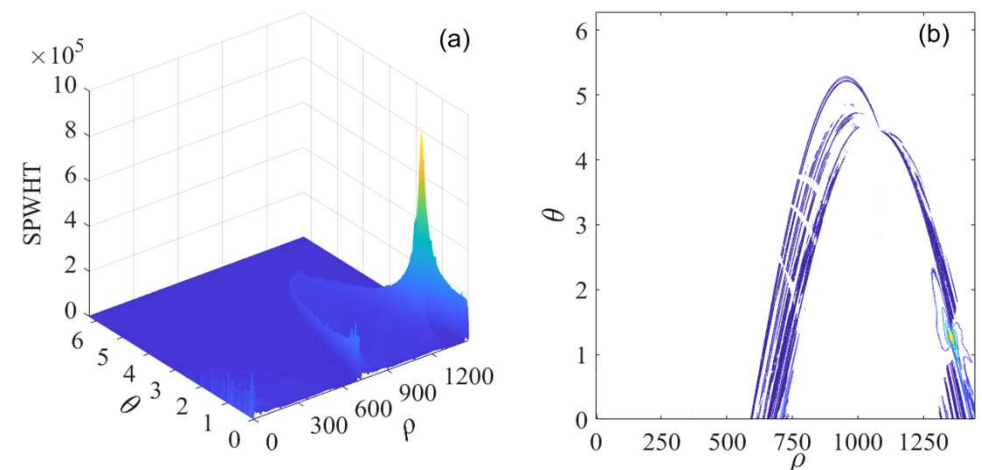


The WHT of GPS L1-C/A signal disturbed by sweep interference.

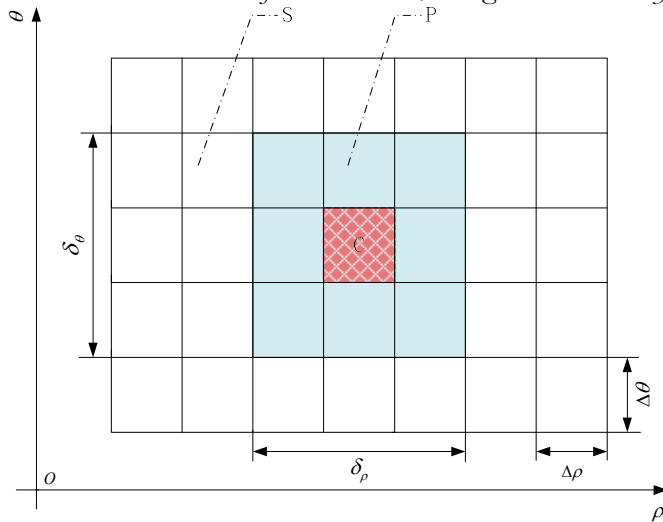
# Smoothed Pseudo WVD–Hough Transform Method



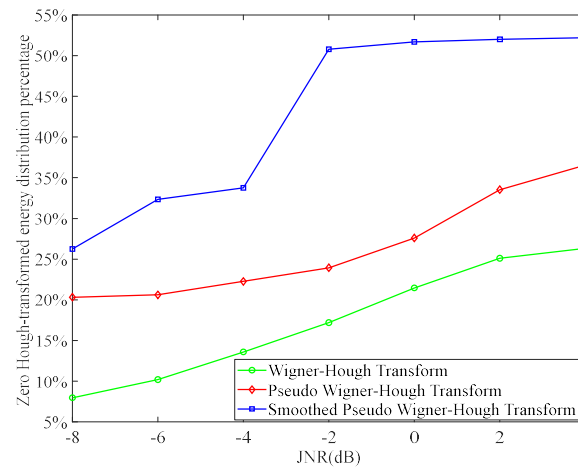
The PWHT of GPS L1-C/A signal disturbed by the sweep interference.



The SPWHT of GPS L1-C/A signal disturbed by the sweep interference.



The zero Hough-transformed energy percentage metrics of the Hough transform techniques.



- Kewen Sun\*, Baoguo Yu, Mireille Elhajj, Washington Yotto Ochieng, etc, A Novel GNSS Interference Detection Method based on Smoothed PseudoWigner-Hough Transform, *Sensors*, 2021, 21(13):4306
- Kewen Sun\*, Jiang Guo, A Novel Interference Detection based on Wigner-Hough Transform for GNSS Receivers, *ION GNSS+ 2019*, Miami, Florida, USA, September 16-20, 2019
- Kewen Sun, A GNSS Receiver Interference Detection Method based on Smooth Pseudo Wigner-Hough Transform, **Patent Application Number: 20241122405.2**

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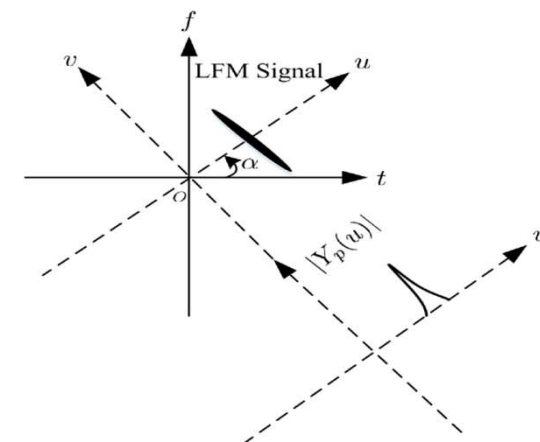


# GNSS Anti-Interference using FrFT Filtering

## Fractional Fourier Transform (FrFT)

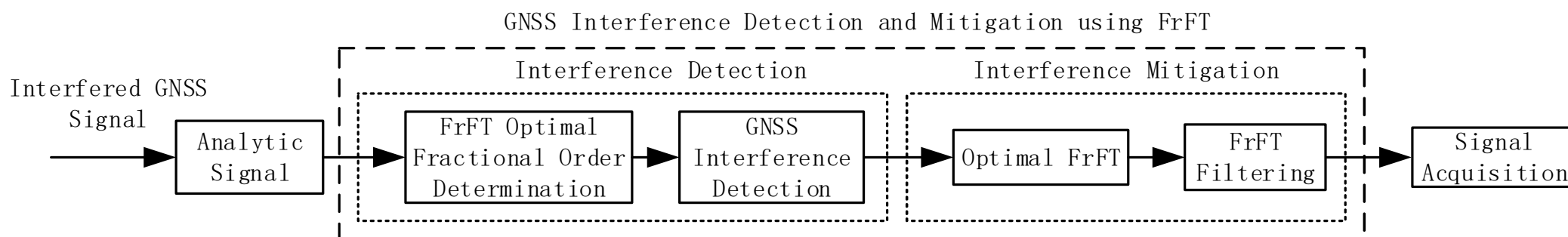
$$Y_p(u) = F_p(u) = \int_{-\infty}^{+\infty} y_a(t) K_p(u, t) dt$$

$$K_p(u, t) = \begin{cases} A_\alpha e^{j\pi[(t^2+u^2)\cdot\cot\alpha-2tu\cdot\csc\alpha]}, & \alpha \neq k\pi \\ \delta(t-u), & \alpha = 2k\pi \\ \delta(t+u), & \alpha = (2k+1)\pi \end{cases}$$



TF distribution of LFM signal and its FrFT projection to the fractional domain.

## GNSS Anti-Interference by using FrFT

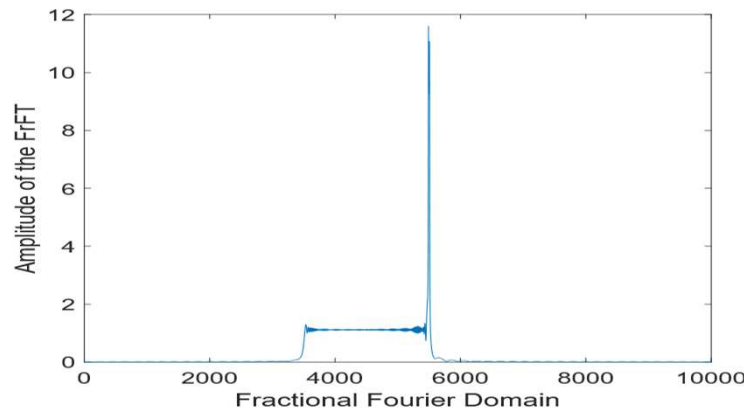


- Kewen Sun\*, Mireille Elhajj, Washington Yotto Ochieng, A GNSS Anti-interference Method based on Fractional Fourier Transform, *IEEE Transactions on Aerospace and Electronic Systems*, 2024, 60(5): 5636 - 5650
- Kewen Sun, A GNSS Receiver Anti-interference Method based on FrFT Filtering, Patent Application Number: 2024111168052

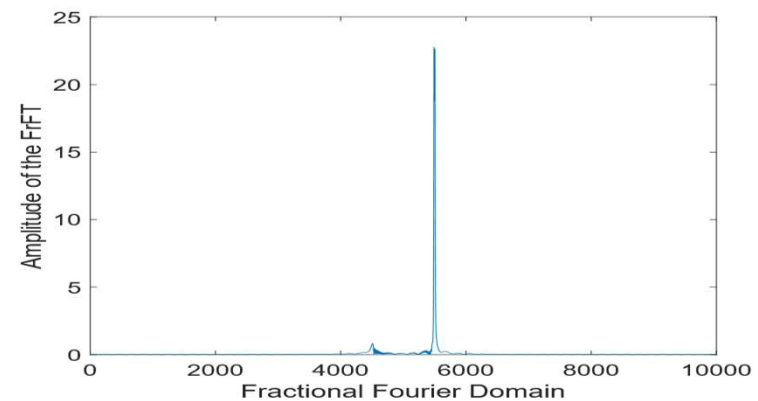
# GNSS Anti-Interference using FrFT Filtering

## Signal Preprocessing by using Hilbert Transform

- ✓ A real LFM interfering signal can be decomposed into the sum of two complex signals, which makes the FrFT of the GNSS interfering signal have a different optimal fractional order. When the FrFT of the interfering signal is observed in the fractional Fourier domain using the optimal fractional order, only the energy of one complex signal is strengthened, the energy of the other complex signal is scattered.



*The FrFT of a real LFM signal in the fractional order Fourier domain.*



*The FrFT of a real LFM signal after Hilbert transform in the fractional order Fourier domain.*

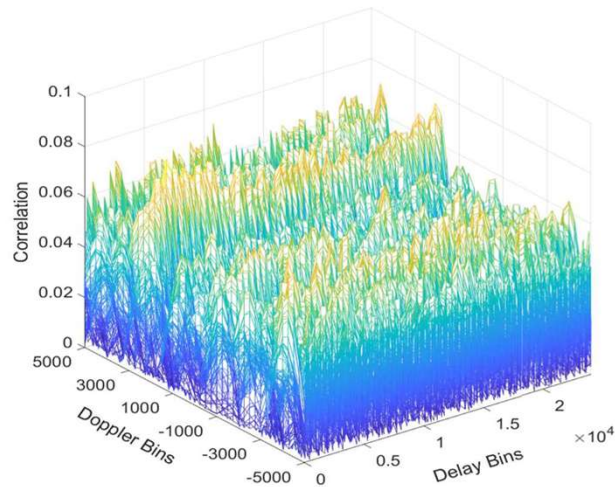
- ✓ When the real LFM signal is preprocessed by the Hilbert transform, **the analytic form of the real LFM signal presents a good property of energy concentration in the fractional domain**, which is beneficial to the filtering or removal of the GNSS interfering signal.

# GNSS Anti-Interference using FrFT Filtering

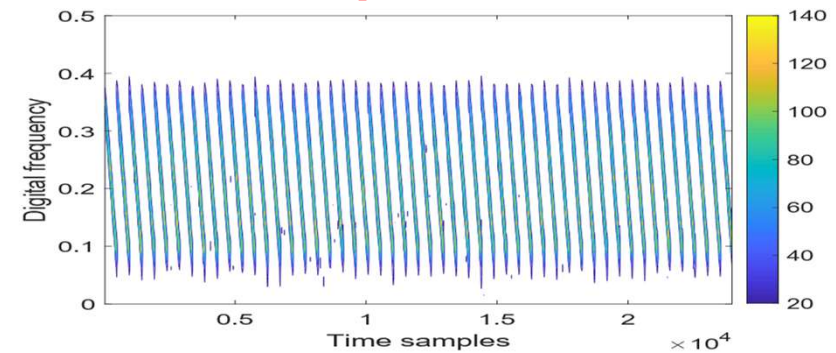
## GNSS Anti-Interference Experimental Parameters Setting

### Parameters in GNSS Anti-interference Experiment

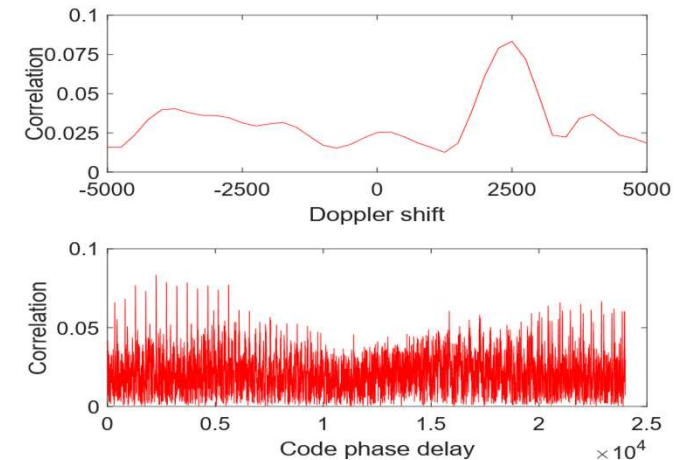
| Parameter                                     | Value    |
|-----------------------------------------------|----------|
| Carrier-to-Noise Ratio, $C/N_0$               | 46 dB-Hz |
| Jamming-to-Noise Ratio, $JNR$                 | 2 dB     |
| Doppler shift, $f_d$                          | 3 kHz    |
| Coherent integration time                     | 1 ms     |
| Left sweep frequency deviation, $\Delta f_0$  | 4.5 MHz  |
| Right sweep frequency deviation, $\Delta f_1$ | -2.5 MHz |
| Sweep period                                  | 0.02 ms  |



The cross ambiguity function evaluated using the interfered GPS L1-C/A signal



Time frequency evolution of the received original interfered GPS L1-C/A signal

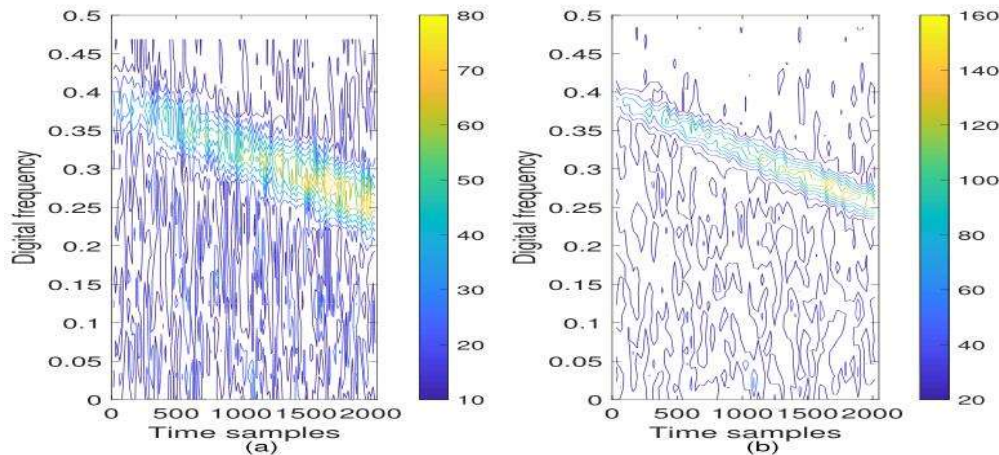


The Doppler shift and code phase delay estimated using the original interfered GPS L1-C/A signal - 24 -

# GNSS Anti-Interference using FrFT Filtering

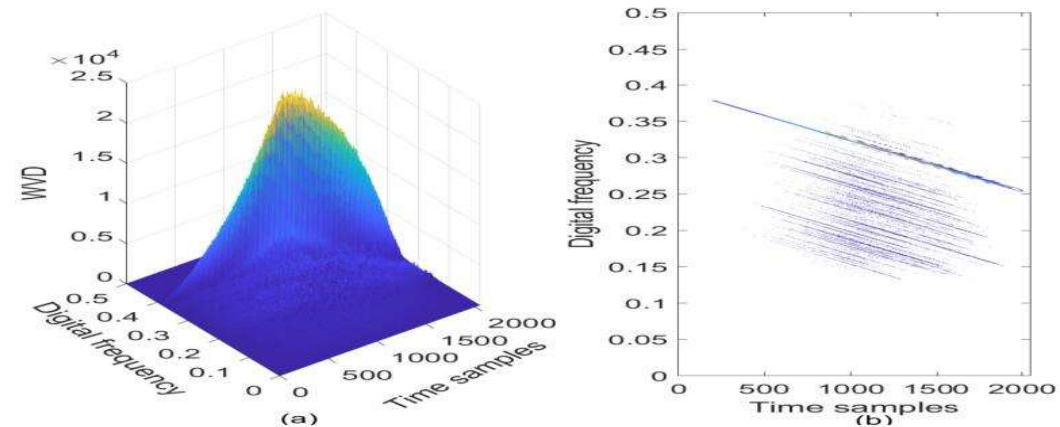
## State of the Art TF Analysis Techniques

✓ TF resolution trade-off problem of the spectrogram



The spectrogram of the interfered GPS L1-C/A signal, adopting a Hamming window function.  $C/N_0 = 46$  dB-Hz,  $JNR = -4$  dB. (a) window size = 31; (b) window size = 63.

✓ The cross-term problem existed in the Wigner-Ville distribution



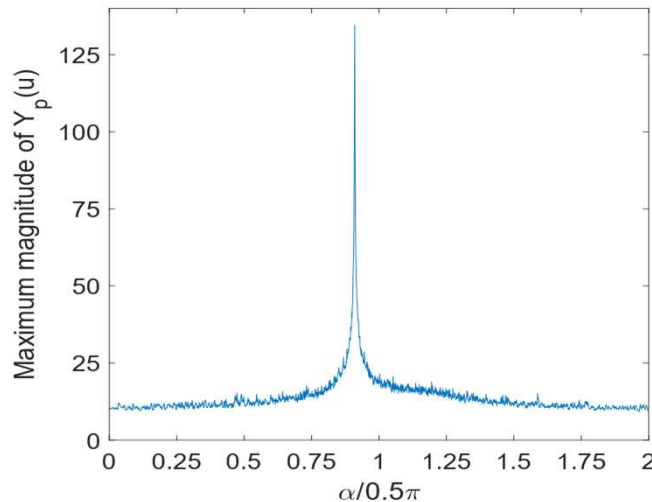
The WVD of the interfered GPS L1-C/A signal.  $C/N_0 = 46$  dB-Hz,  $JNR = -4$  dB. (a) WVD; (b) contour of WVD.

# GNSS Anti-Interference using FrFT Filtering

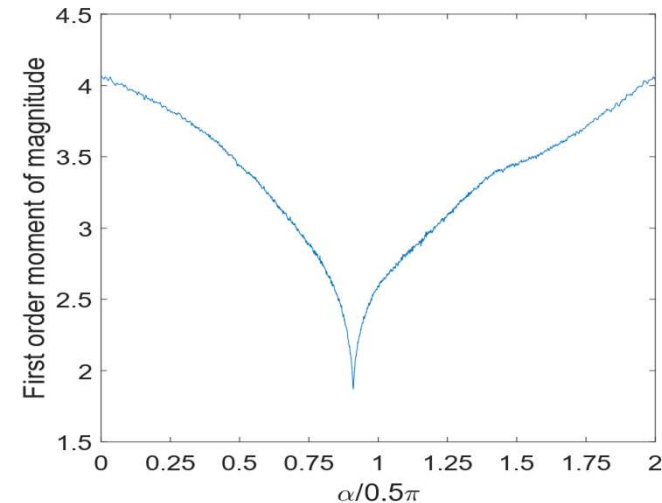
## Optimal Fractional Order Determination

- ✓ In order to address the problem of inefficient search methods and unreliable interference detection and mitigation of the current techniques. An efficient optimal fractional order determination model **First Order Moment of Magnitude (FOMM) of the FrFT** has been developed to reduce the computational complexity, expressed as:

$$\overline{Y_p(u)} = \frac{1}{N} \sum_{u=1}^N |Y_p(u)| \Rightarrow p_{opt,FOMM} = \arg \min_p \overline{Y_p(u)}$$



The maximum magnitude (MM) of the FrFT of the interfered GPS L1-C/A signal.

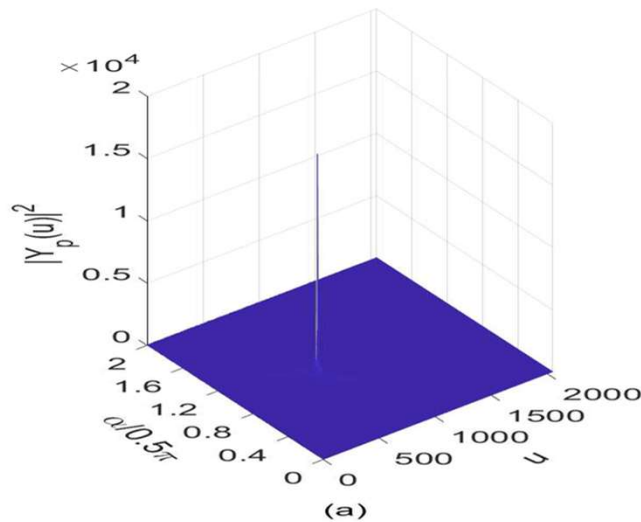


The first order moment of magnitude (FOMM) of the interfered GPS L1-C/A signal. -26-

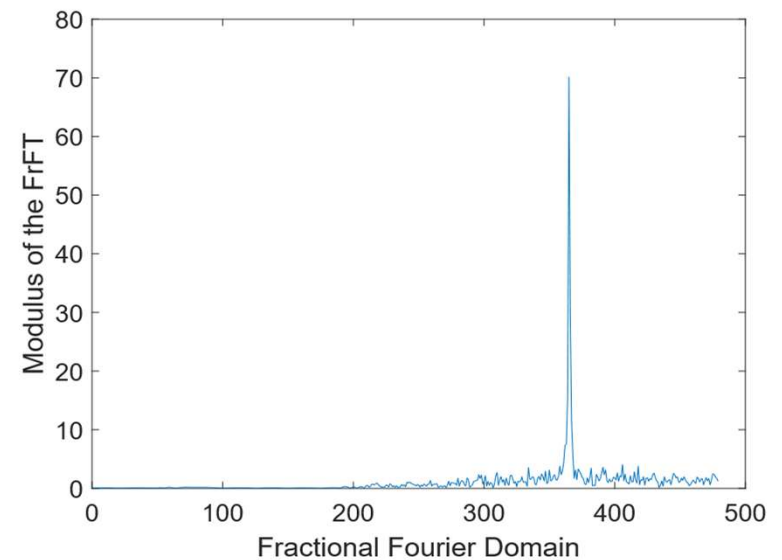
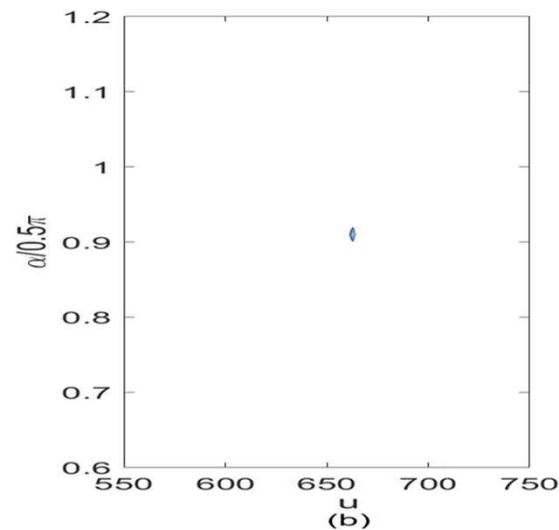
# GNSS Anti-Interference using FrFT Filtering

## FrFT based GNSS Interference Detection

- ✓ When the FrFT is used in interference detection, **the problem of detecting the sweep interference in the TF plane is converted to the problem of determining the FrFT energy peak location in the u-p plane.**



The FrFT based interference detection for the interfered GPS L1-C/A signal.

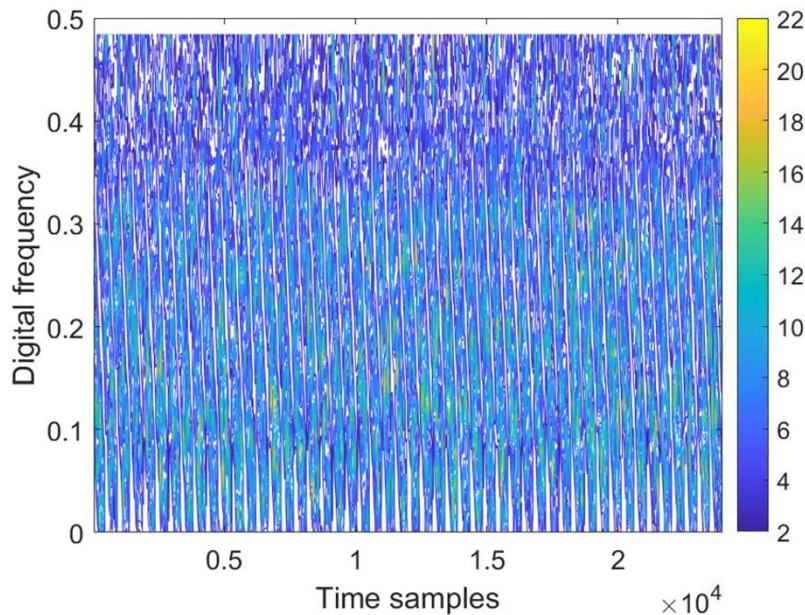


The modulus of the FrFT of the received original interfered GPS L1-C/A signal in a sweep period.

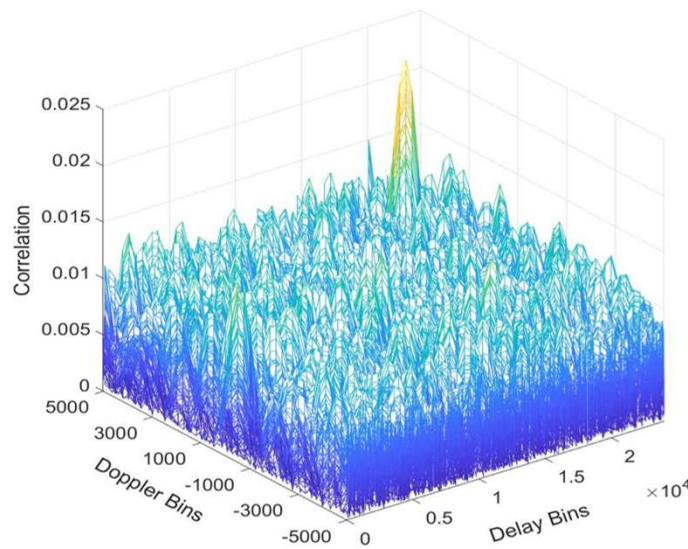
# GNSS Anti-Interference using FrFT Filtering

## FrFT Filtering

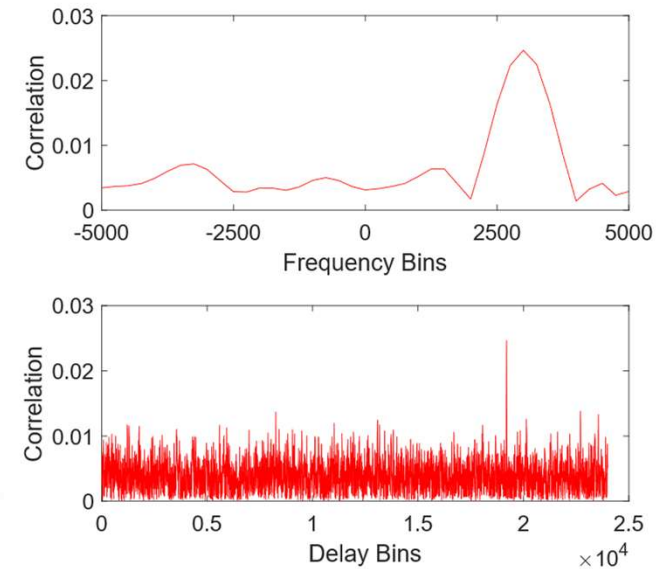
- ✓ When the FrFT filtering is used in GNSS interference, the sweep interfering components can be effectively removed.



*The time-frequency evolution of the FrFT-filtered GPS L1-C/A signal.*



*The cross ambiguity function evaluated using the interfered GPS L1-C/A signal processed adopting the FrFT filtering method. The sweep period of the sweep interference is 0.02 ms, and the coherent integration time is 1 ms in the signal acquisition process.*

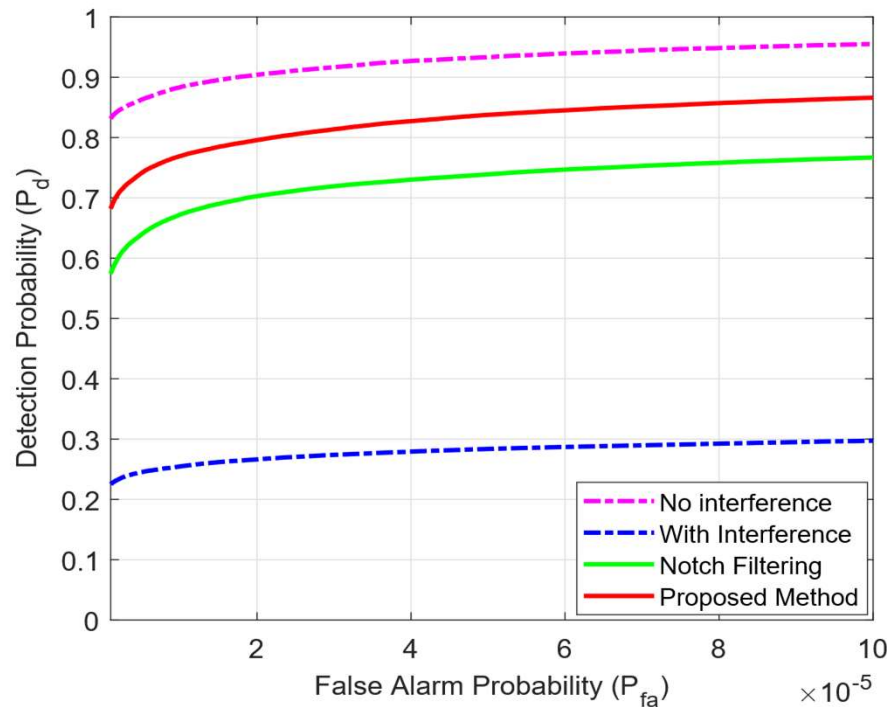


*The Doppler shift and code phase delay estimated using the interfered GPS L1-C/A signal processed adopting the FrFT filtering method.*

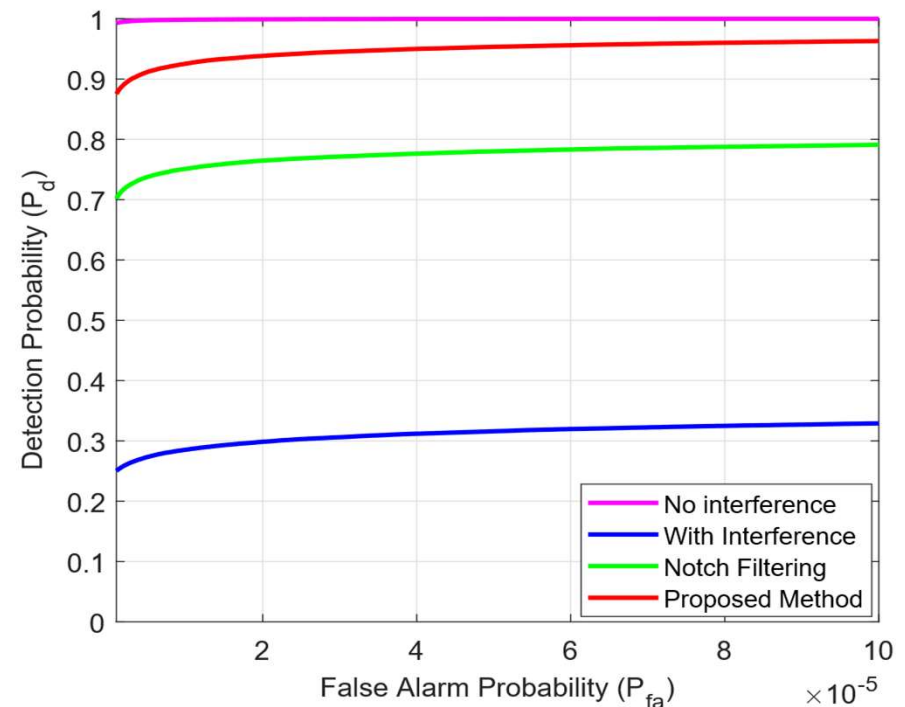
# GNSS Anti-Interference using FrFT Filtering

## GNSS Anti-Interference Performance Evaluation

### Receiver Operating Characteristic (ROC) Comparison



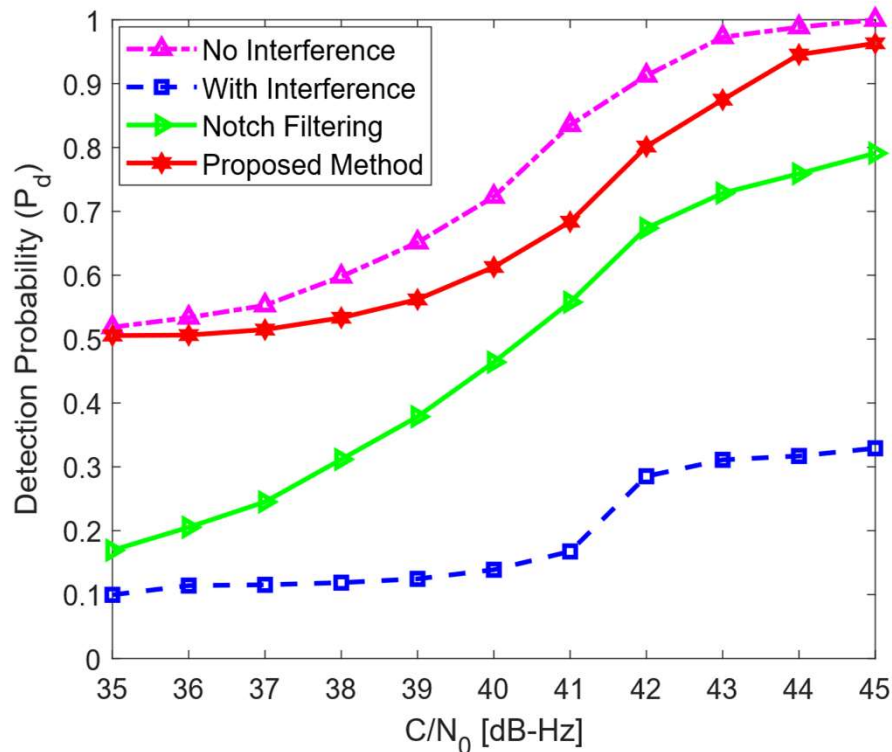
The ROC curves of the GPS L1-C/A signal in different signal scenarios. The coherent integration time is 2 ms,  $C/N_0 = 40$  dB-Hz,  $J/NR = 20$  dB.



The ROC curves of the GPS L1-C/A signal in different signal scenarios. The coherent integration time is 1 ms,  $C/N_0 = 45$  dB-Hz,  $J/NR = 10$  dB.

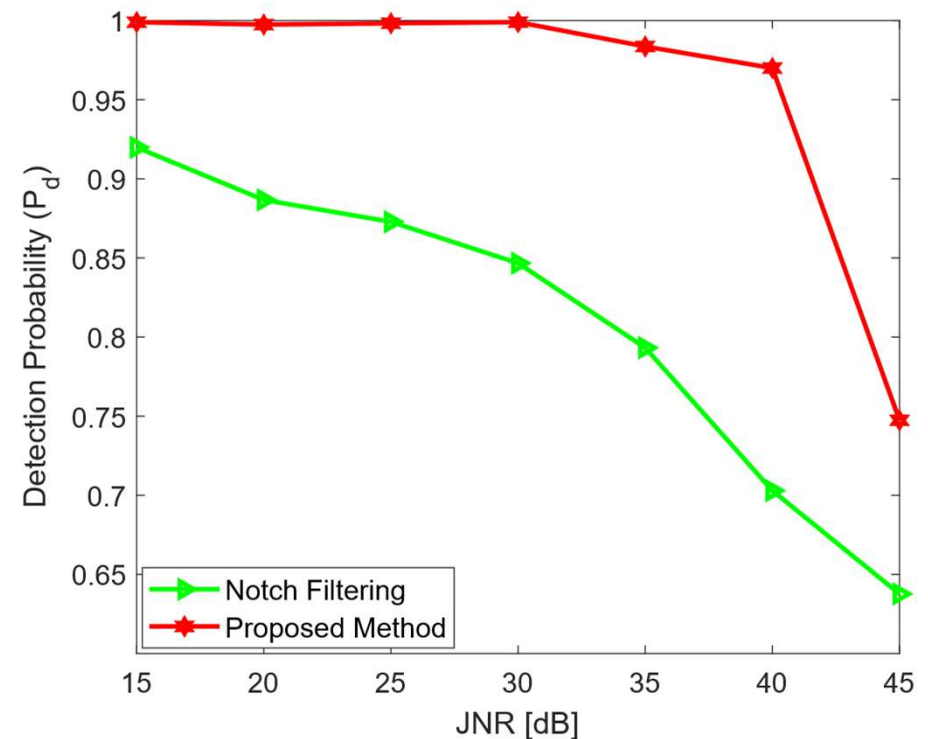
# GNSS Anti-Interference using FrFT Filtering

## ■ SNR Curve Comparison



The SNR curves of the GPS L1-C/A signal in different signal scenarios. The coherent integration time is 1 ms in the signal acquisition.

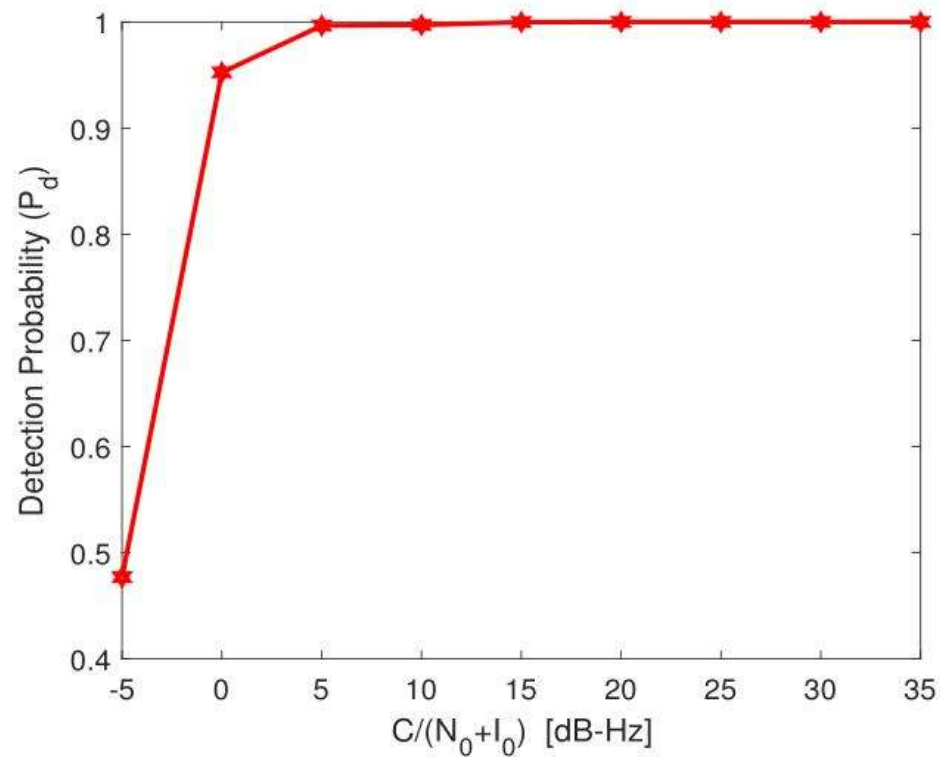
## ■ Jamming Tolerance Analysis



The detection probabilities in the acquisition of GPS L1-C/A signal with  $C/N_0$  equal to 45 dB-Hz by using two-pole notch filter and FrFT techniques respectively in different interfering scenarios. The coherent integration time is 2 ms.

# GNSS Anti-Interference using FrFT Filtering

## ■ $C/(N_0+I_0)$ Curve Analysis



*The  $C/(N_0+I_0)$  curve of the FrFT based anti-interference method using GPS L1-C/A signal in different interfering scenarios. The coherent integration time is 2 ms in the signal acquisition.*

# Outline

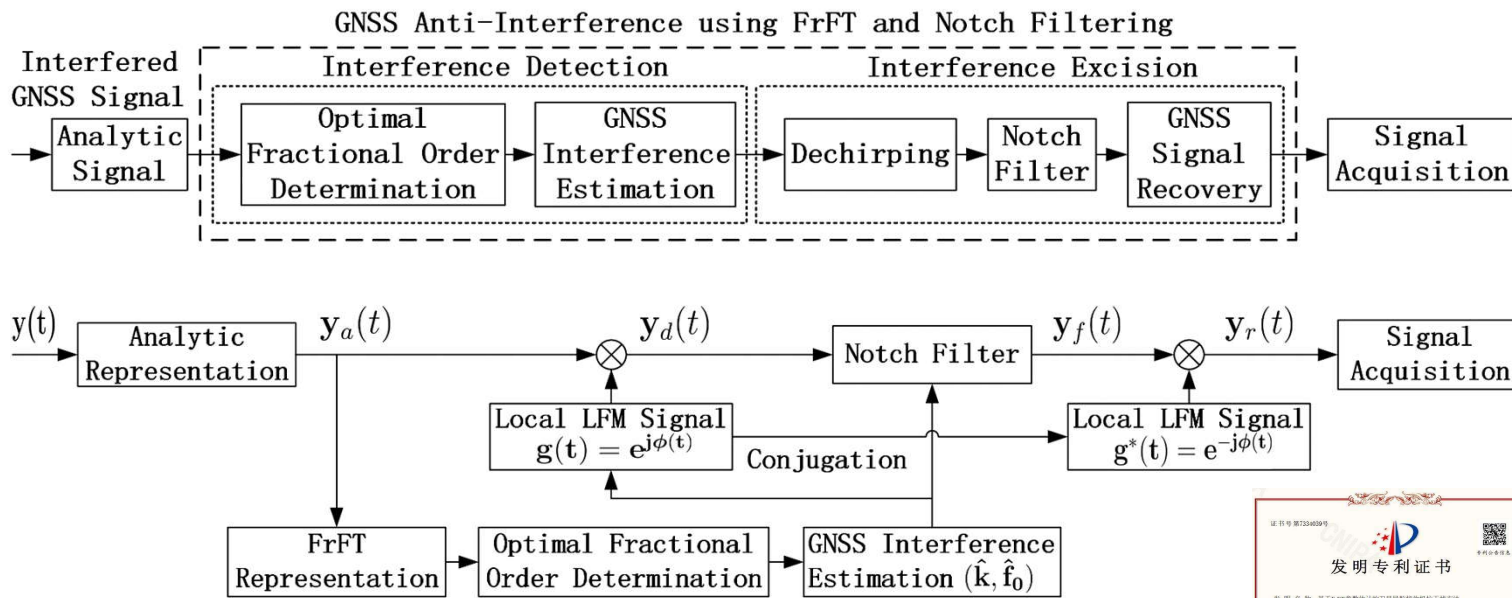
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- **Satellite Navigation Countermeasures**
- **GNSS Interference Detection based on TF Analyses**
- **GNSS Interference Detection based on Hough Transform**
- **GNSS Anti-Interference based on FrFT**
- **GNSS Anti-Interference based on FrFT & Notch Filtering**



# GNSS Anti-Interference using FrFT and Notch Filtering

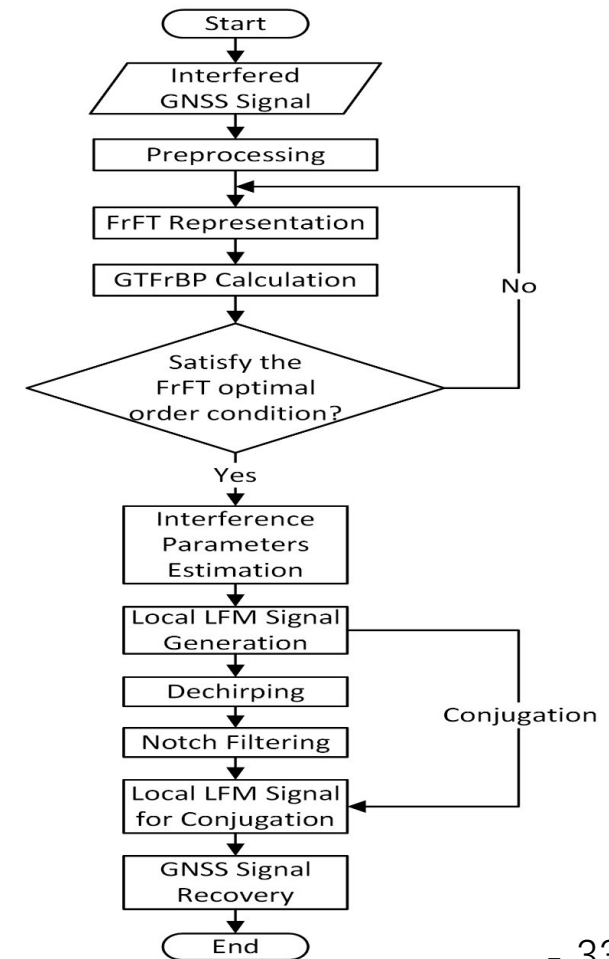
## FrFT & Notch Filtering based Anti-Interference Scheme



- **Kewen Sun\***, Baoguo Yu, Liangfeng Xu, Mireille Elhajj, Washington Yotto Ochieng, A Novel GNSS Anti-Interference Method using Fractional Fourier Transform and Notch Filtering, *IEEE Transactions on Instrumentation & Measurement*, DOI: 10.1109/TIM.2024.3451600, 2024
- **Kewen Sun**, Haijun Huang, Yin Wang, Cheng Wu, Cong Wang, Jun Wang, A GNSS Receiver Anti-interference Method based on FrFT Parameters Estimation, **Patent Number: ZL 2022 1 0248955.3**



Patent Certificate



# GNSS Anti-Interference using FrFT and Notch Filtering



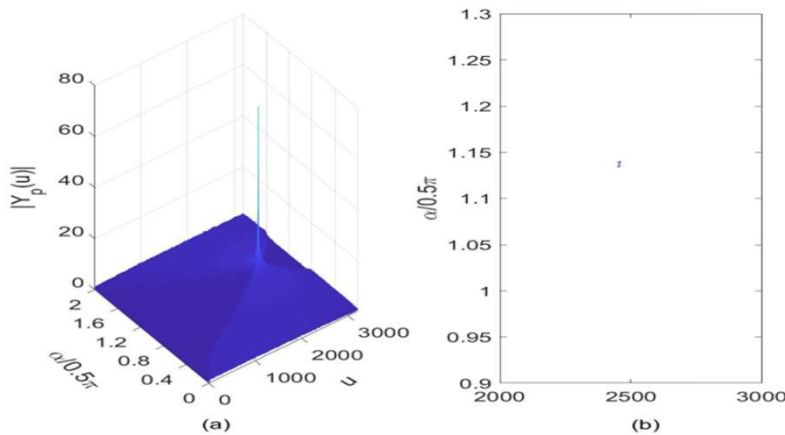
## Determination of the Optimal Fractional Order of FrFT

### TFrBP Model

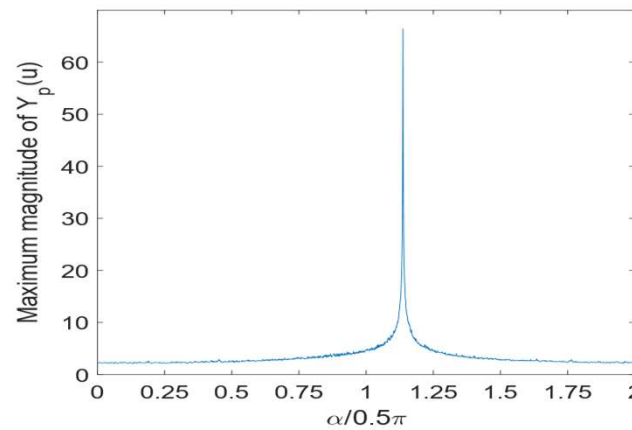
✓ The criterion of **time fractional bandwidth product (TFrBP)** has been developed, given as:

$$TFrBP\{y(t), F_p[y(t)]\} = T_y \cdot FrB_y \Rightarrow p_{opt, GTFrBP} = \arg \min_p TFrBP\{y(t), F_p[y(t)]\}$$

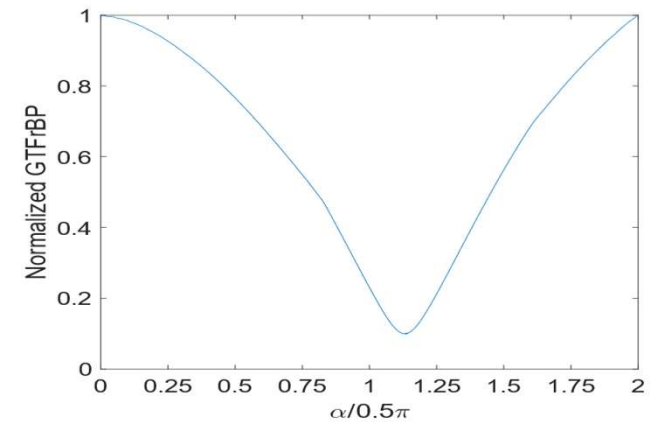
where  $T_y = \frac{[\int (t-\eta_t)^2 |y(t)|^2 dt]^{1/2}}{\|y(t)\|}$   $FrB_y = \frac{[\int (u-\eta_u)^2 |Y_p(u)|^2 du]^{1/2}}{\|Y_p(u)\|}$   $\eta_t = \frac{[\int t |y(t)|^2 dt]^{1/2}}{\|y(t)\|}$   $\eta_u = \frac{[\int u |Y_p(u)|^2 du]^{1/2}}{\|Y_p(u)\|}$



The FrFT based interference detection for the GPS L1-C/A signal in the presence of sweep interference.  $C/N_0 = 45$  dB-Hz,  $JNR = 3$  dB.



The maximum magnitude of the FrFT of the GPS L1-C/A signal in the presence of sweep interference.

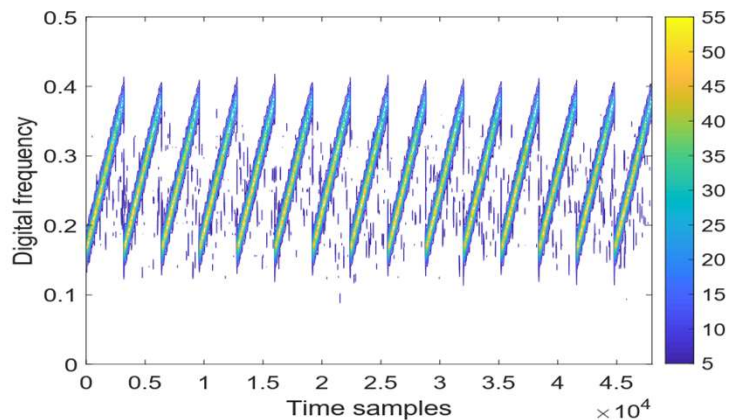


The normalized GTFrBP of the GPS L1-C/A signal in the presence of sweep interference.

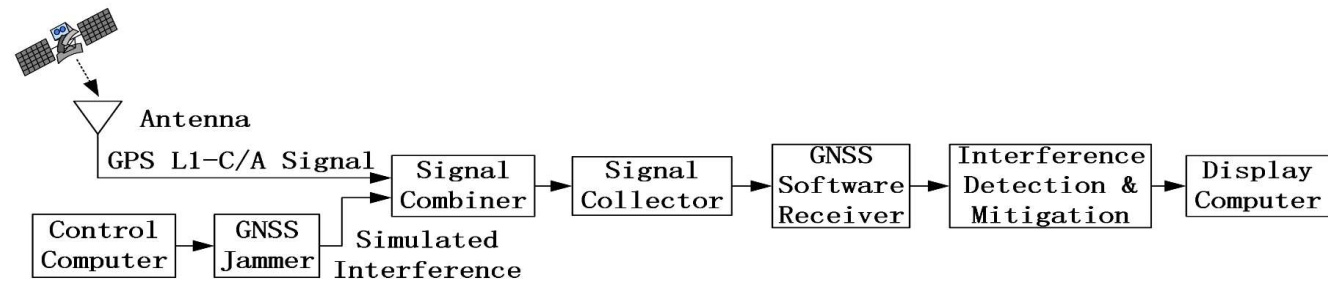
# GNSS Anti-Interference using FrFT and Notch Filtering

## GNSS Anti-interference Experimental Tests

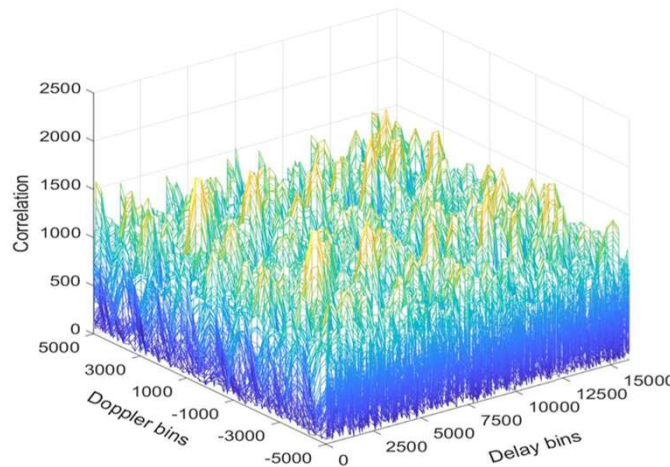
- ✓ When no anti-interference countermeasure is activated, failed result of acquiring the GPS L1-C/A signal (PRN 3) is shown, with no emerging CAF peak found in the search space.



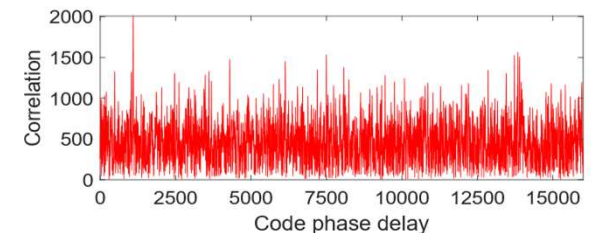
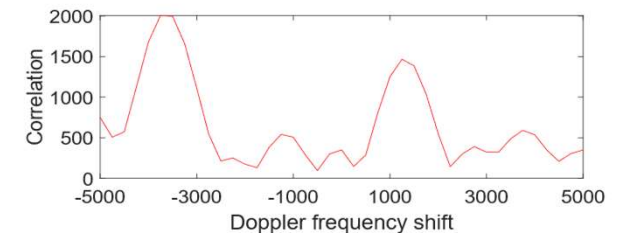
The spectrogram of the interfered GPS L1-C/A signal with the duration of 3 ms.



The experimental scheme of the proposed GNSS anti-interference method.



The cross-ambiguity function of the interfered GPS L1-C/A signal (PRN 3): 1 ms coherent integration time and no GNSS interference mitigation.

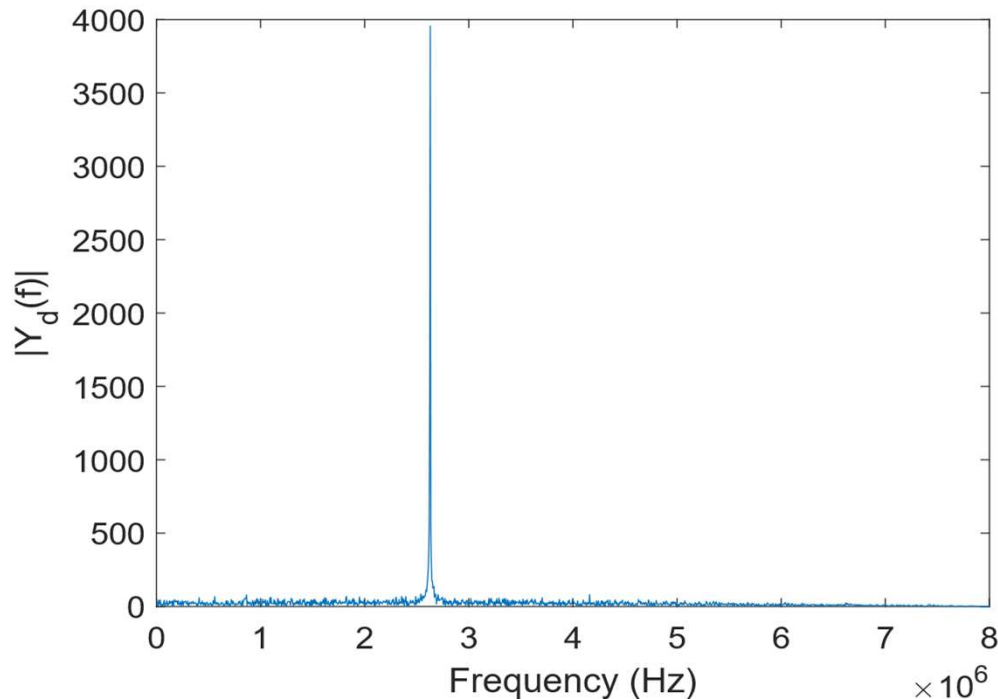


The estimated Doppler shift and code phase delay of the original interfered GPS L1-C/A signal (PRN 3) when anti-interference is not activated.

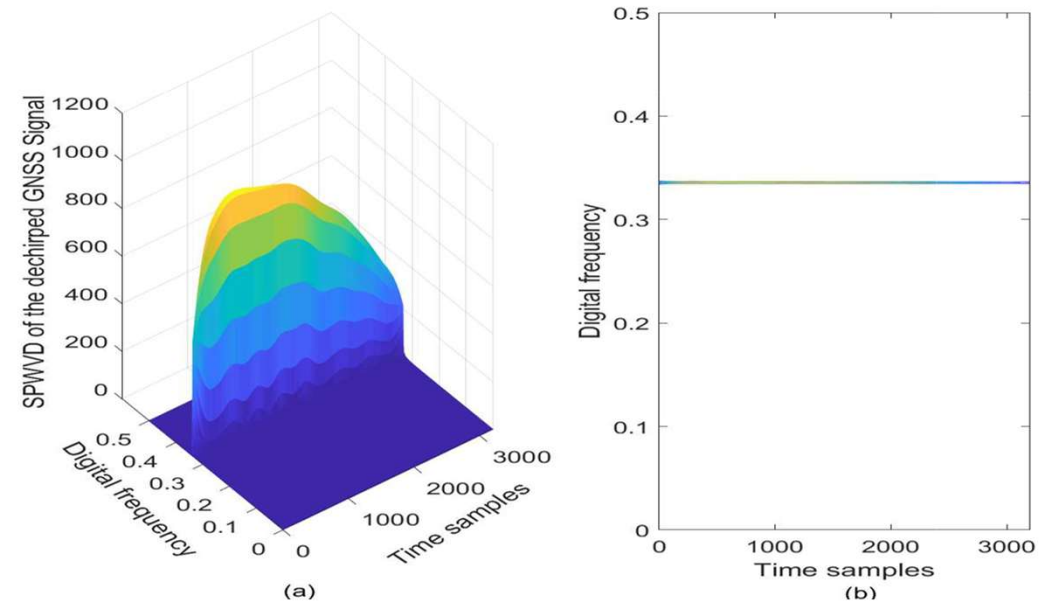
# GNSS Anti-Interference using FrFT and Notch Filtering

## ■ Dechirping

- ✓ By using the estimated sweep rate of the sweep interfering signal, a local LFM signal can be constructed to dechirp the originally received GNSS signal.



*The amplitude spectrum of the interfered GPS L1-C/A signal after dechirping.*

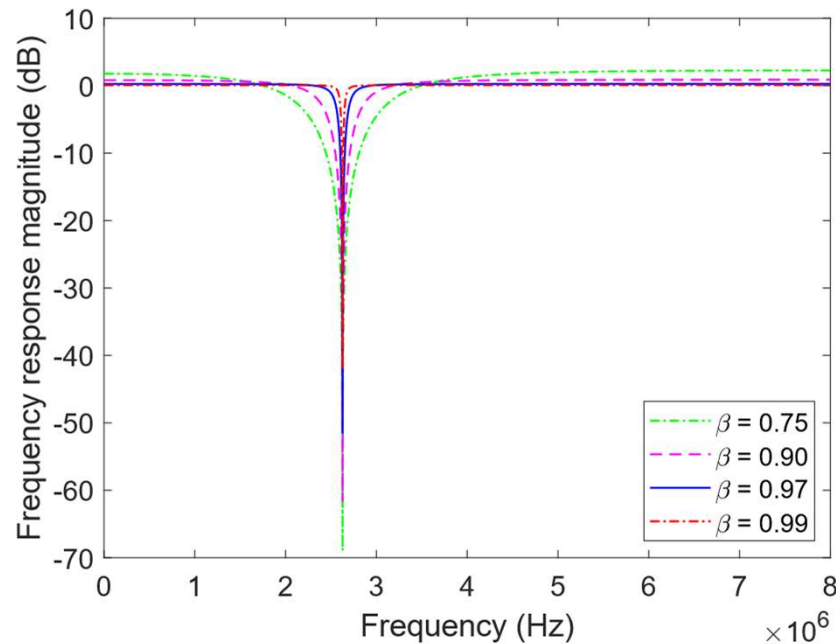


*The SPWVD of the interfered GPS L1-C/A signal after dechirping: (a) the SPWVD; (b) the SPWVD's contour.*

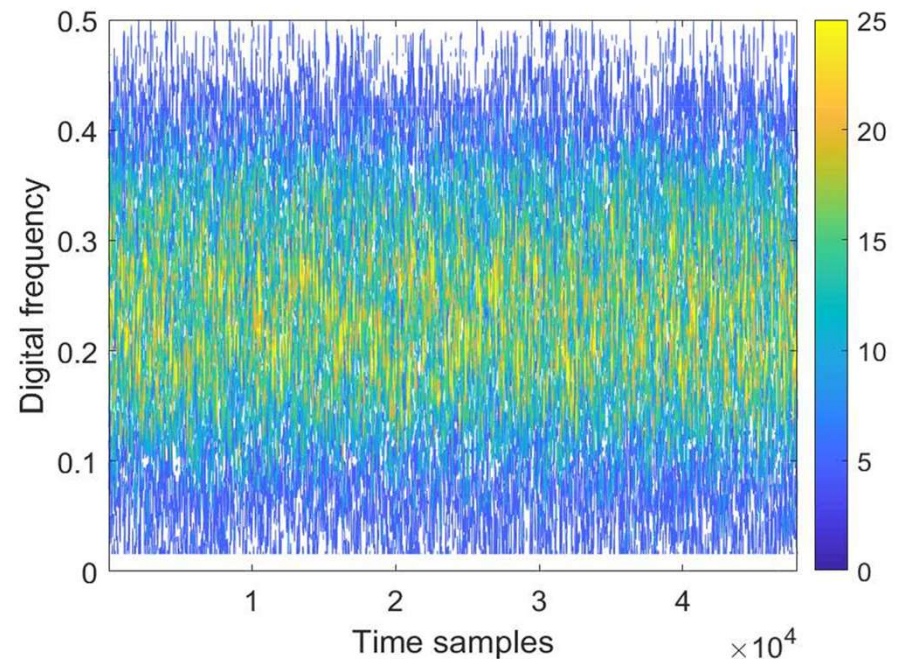
# GNSS Anti-Interference using FrFT and Notch Filtering

## ■ Notch Filtering

- ✓ Based on the starting frequency estimate of the GNSS sweep interfering signal obtained by adopting the proposed method, **a single frequency notch filter is designed to achieve clean filtering of the interference.**



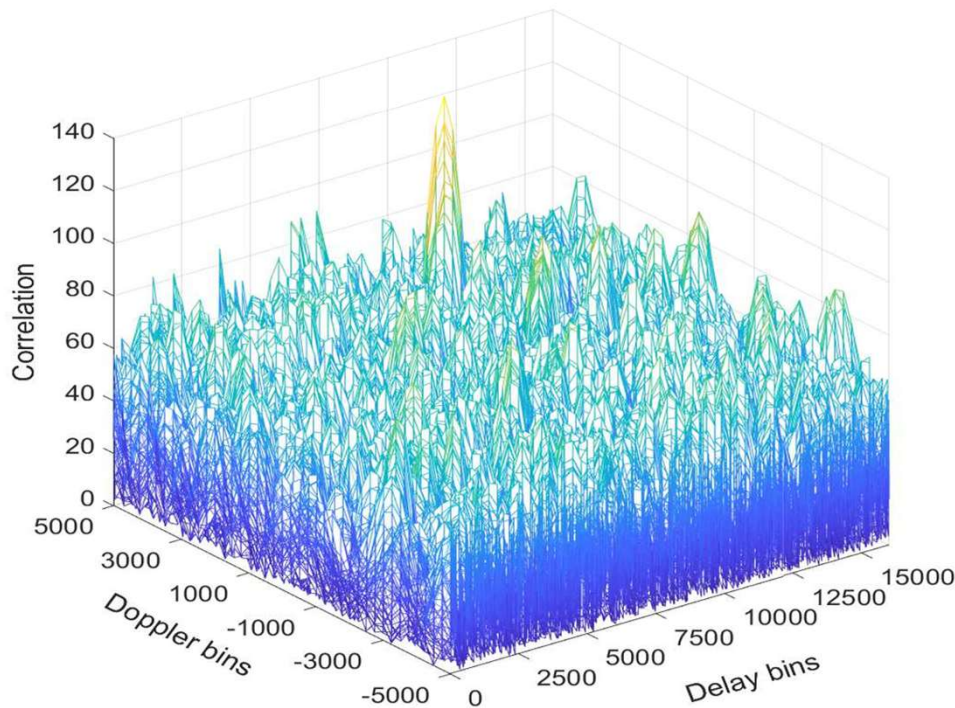
*The magnitude frequency response curves of the transfer function of the notch filter using different filter pole radius values.*



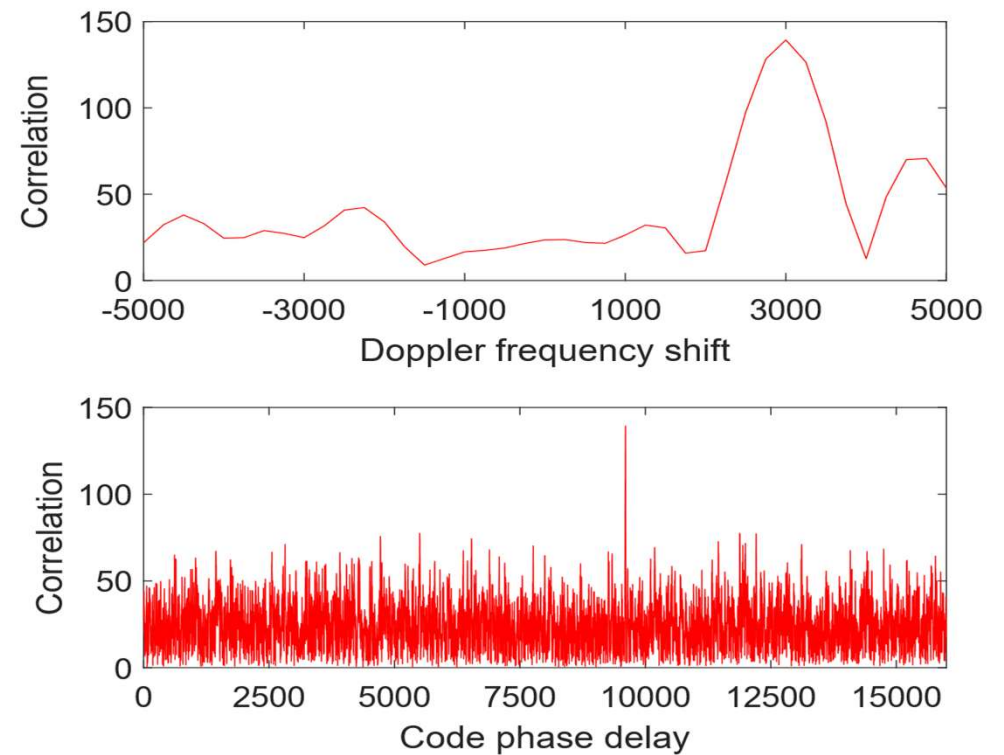
*The spectrogram of the recovered GPS L1-C/A signal with the duration of 3 ms after interference suppression.*

# GNSS Anti-Interference using FrFT and Notch Filtering

## ■ Acquisition Results after Interference Suppression



*The cross ambiguity function of the interfered GPS L1-C/A signal using the proposed FrFT and notch filtering based GNSS interference detection and mitigation method: 1 ms coherent integration time.*

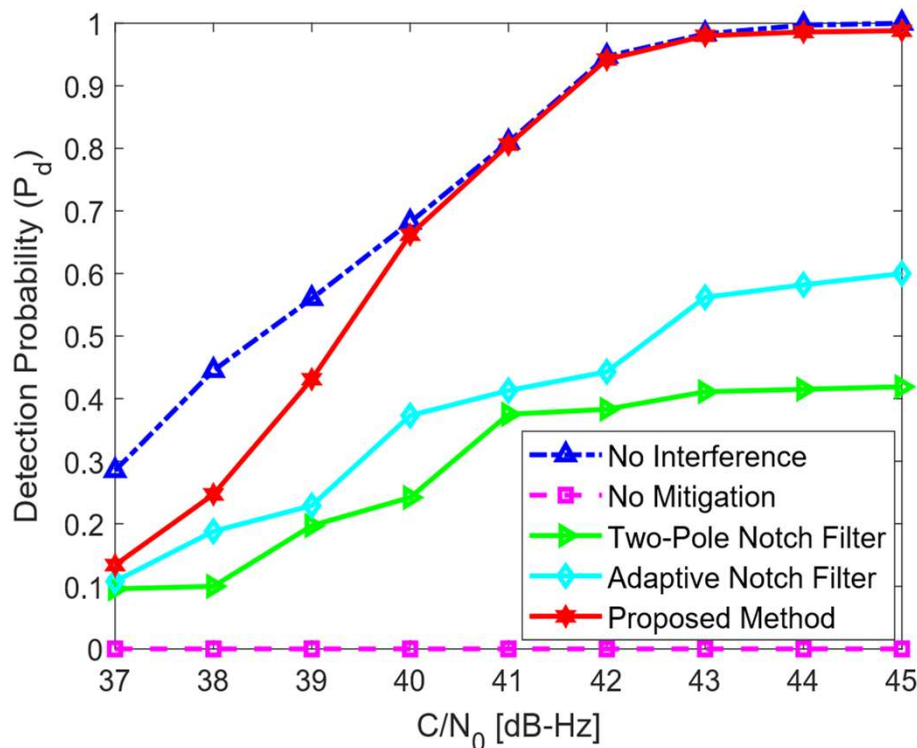


*The Doppler shift and code phase delay estimates of the GPS L1-C/A signal with the sweep interference by adopting the proposed GNSS anti-interference method.*

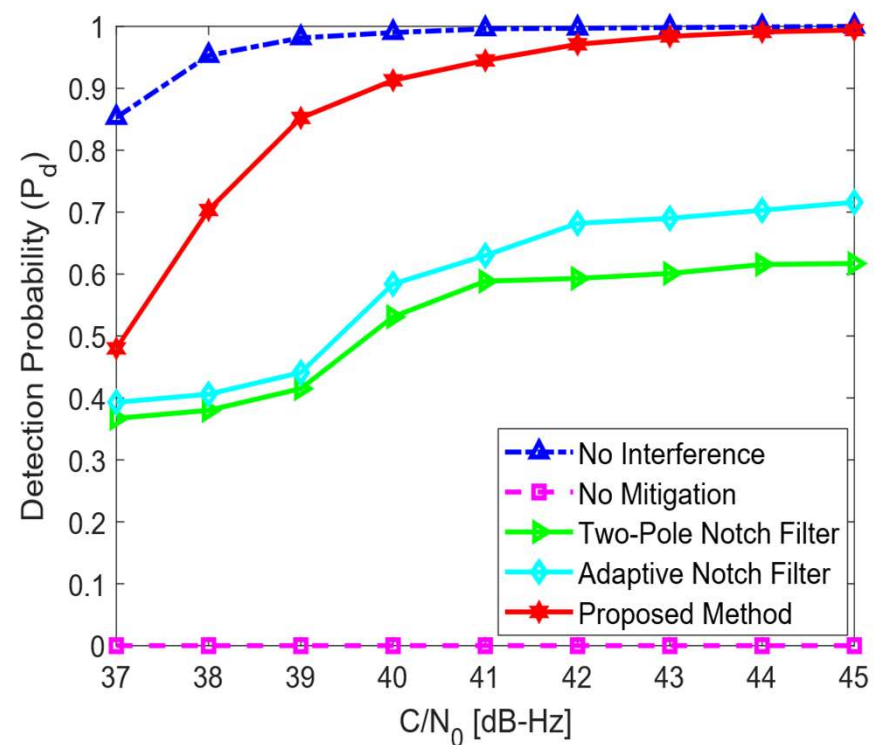
# GNSS Anti-Interference using FrFT and Notch Filtering

## Performance Evaluation

### C/N<sub>0</sub> Curve Comparison



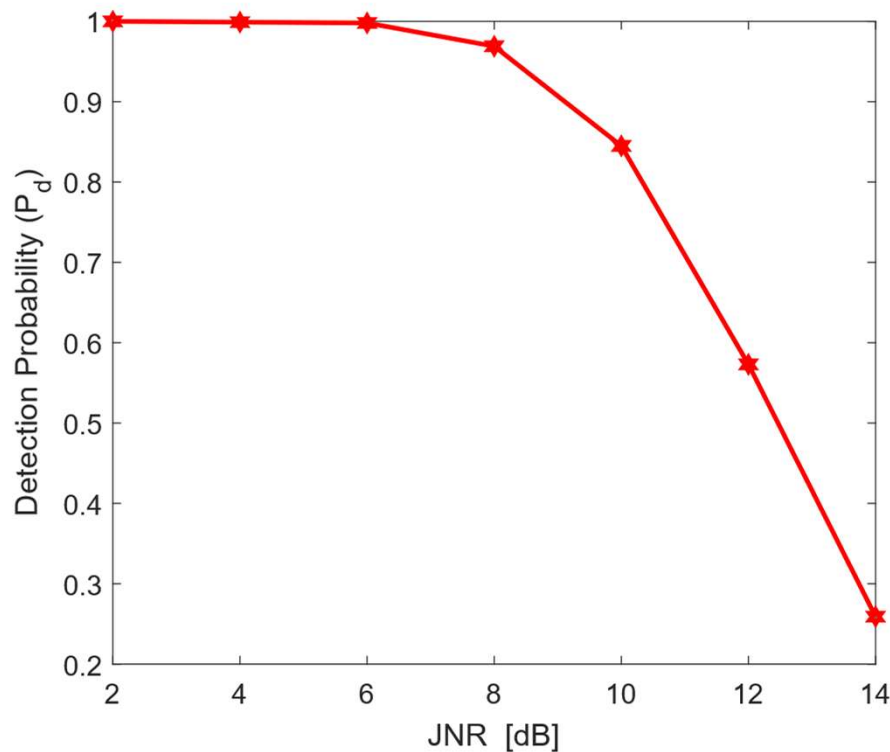
The  $C/N_0$  curves of the GPS L1-C/A signal acquisition procedure, with the coherent integration time being 2 ms. The JNR value is set to 2 dB when the sweep interference is present in the received GPS L1-C/A signal.



The  $C/N_0$  curves of the GPS L1-C/A signal acquisition procedure, with the coherent integration time being 4 ms. The JNR value is set to 5 dB when the sweep interference is present in the received GPS L1-C/A signal.

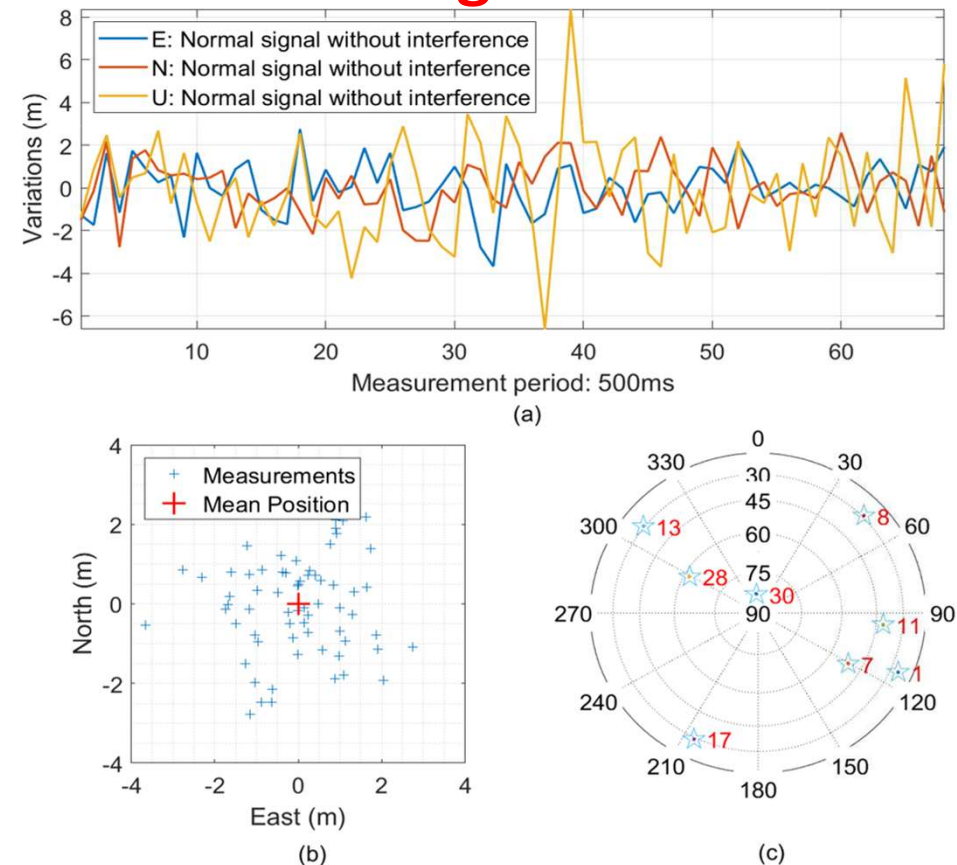
# GNSS Anti-Interference using FrFT and Notch Filtering

## ■ Jamming Tolerance Analysis



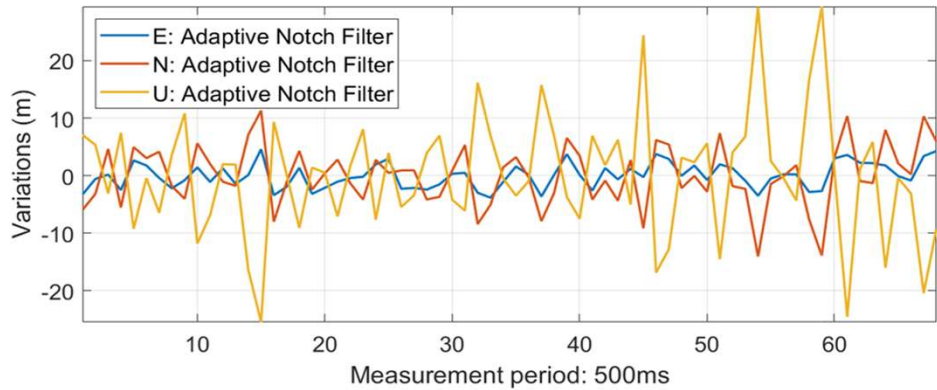
The detection probabilities in the acquisition of GPS L1-C/A signal with the  $C/N_0$  value being 45 dB-Hz by using the proposed method in different interfering scenarios. The coherent integration time is 2 ms.

## ■ Positioning Results

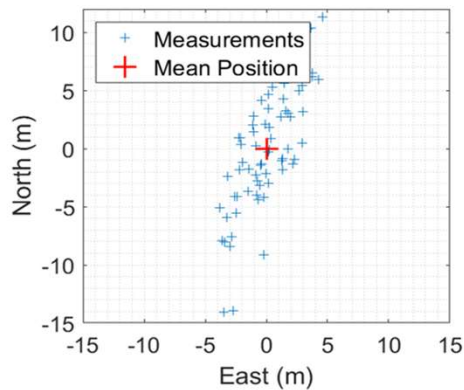


The positioning results **for the normal situation without interference in the real GPS L1-C/A signal**. (a) The variations in coordinates within the UTM system; (b) The positioning errors in the UTM system; (c) The sky plot displaying the satellite constellation.

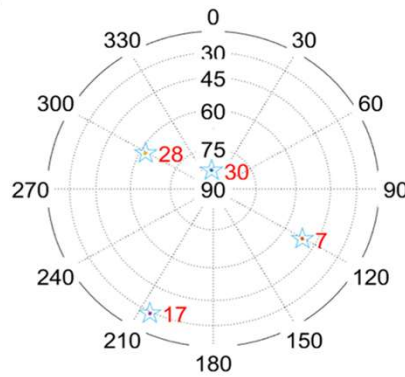
# GNSS Anti-Interference using FrFT and Notch Filtering



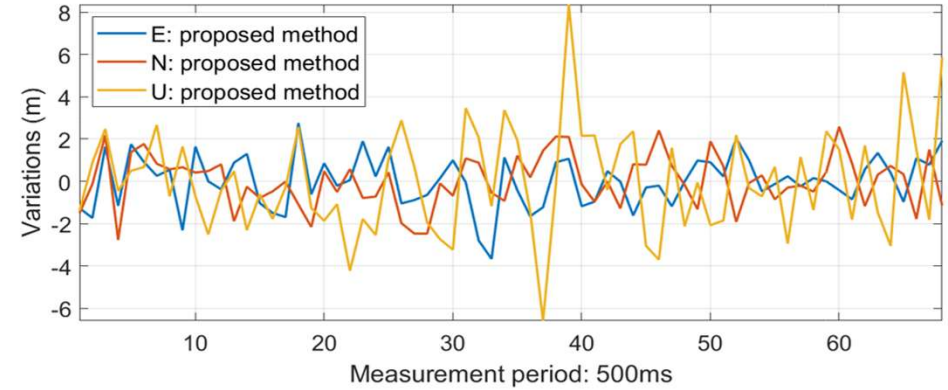
(a)



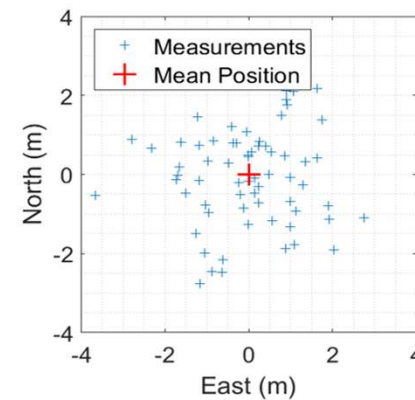
(b)



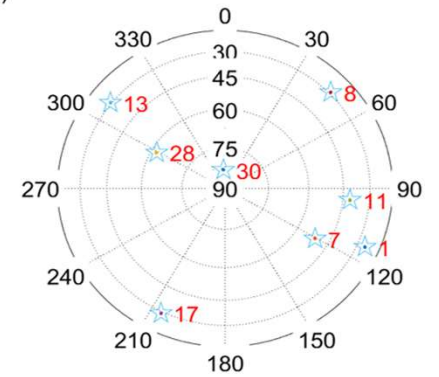
(c)



(a)



(b)



(c)

The positioning results obtained using **the adaptive notch filter technique** in the presence of sweep interference in the real GPS L1-C/A signal, with a sweep period of 0.2 ms and a jammer-to-noise ratio (JNR) value of 3 dB. (a) The variations in coordinates within the UTM system; (b) The positioning errors in the UTM system; (c) The sky plot displaying the satellite constellation.

The positioning results by using **the proposed method**: the sweep interference is within the real GPS L1-C/A signal, with the sweep period being 0.2 ms and JNR value equal to 3 dB. (a) The variations in coordinates within the UTM system; (b) The positioning errors in the UTM system; (c) The sky plot displaying the satellite constellation.

Thank you for your  
attention!

